•••POWER
ENGINEERING

Fourth Class

Edition 3.5

Elementary Mechanics and Dynamics

Part A

Unit A-1

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Partner in Education

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A. Ojimelukwe — **NAIT**

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PanGlobal
Partner in Education
1–1339 40 Ave NE
Calgary, Alberta
T2E 8N6

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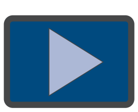
MEDIA 	Media icons show that some form of supporting medium is available. These media may include videos, animations, 2D and 3D models, multi-views of cutaways, simulations, and more. Media elements are available to registered textbook purchasers online via MyPower Study.
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4th Class Edition 3.5 • Part A

UNIT A-1

ELEMENTARY MECHANICS AND DYNAMICS

Unit Introduction

Elementary Mechanics focuses on the behavior of physical objects, particularly when these objects are subject to external forces. The study of mechanics includes statics, where objects placed under load remain stationary; or dynamics, where objects are subject to motion due to external forces. Once there is an understanding of these physical events, they can be interpreted in mathematical form. The mathematical expressions may then be used to solve everyday problems.

The purpose of this unit is to develop confidence describing the physical world, using mathematical expressions.

This unit introduces force, and how it is related to acceleration. It will then use the concept of force to discuss moments, and how moments can be used to solve static based problems.

Work, power, and energy are covered, as well as friction, stress and strain, and power transmission. It will be seen that force is common to understanding work, pressure, stress, and friction. A Power Engineer needs to have a strong understanding of these concepts.

This unit shows how to solve problems relating to:

- static objects
- moving objects
- objects subjected to force
- the result that force may have on the internal resistance of the objects and
- how forces are transmitted from one object to another.

Unit Rationale

It is important for a Power Engineer to have a good understanding of the concept of force; including the relationships between force with work, energy, pressure, and stress. The Power Engineer needs to be able to apply these concepts properly in order to ensure the safe and efficient operation of the plant.

CHAPTER 1

Introduction to Basic Mechanics

Learning Outcome

When you complete this chapter you should be able to:

Apply basic terms and calculations used in the study of mechanics.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Define mass, force, acceleration, velocity, and weight.*
2. *Perform simple calculations involving force, pressure, work, power, and energy.*

Chapter Introduction

Mechanics is a branch of physics that deals with forces and the effect of these forces on bodies at rest and in motion. The science of mechanics is used when studying the forces acting upon:

a) a girder supporting a boiler
b) the leverage exerted in tightening a bolt with a wrench
c) the work done in pumping water to a supply tank
d) the work done in raising an elevator in its shaft

Each of these processes involves the action or movement of forces. There are two branches of mechanics: statics and dynamics.

Statics deals with forces and their effects on rigid bodies at rest. Dynamics deals with motion, and the effects of forces acting upon rigid bodies in motion.

Newton's Laws of Motion

Many of the problems in mechanics deal with forces and motion; therefore, it is important to know and understand the laws of motion. Sir Isaac Newton summarized these laws as follows.

1. Every body will continue in its state of rest, or uniform motion in a straight line, unless an external force acts upon it.
2. The rate of change of motion is proportional to the force producing the change and takes place in the direction in which the force acts.
3. To every action, there is an equal and opposite reaction. When one object exerts a force on a second object, the second object exerts a reactive force of equal magnitude and opposite direction on the first object.

If a large enough force is applied to an object, so that friction is overcome, the object will begin to increase in speed. The larger the force and the lighter the object, the greater the increase in speed will be. This example shows the relationship between force, mass, and acceleration.

This chapter will cover this relationship in detail. It will show how this relationship is expressed mathematically, and how it can be used to solve various problems. The differences between weight and mass will also be discussed.

Concepts including pressure, work power, and energy will be touched upon and covered in detail in the upcoming chapters.

Objective 1

Define mass, force, acceleration, velocity, and weight.

Mass

Mass is the quantity of matter a body contains. The unit of mass in the SI system is kilogram (kg).

Force

Force may be defined as any action on a body that tends to change its size, shape, or its state of rest or motion. In the SI system, the unit of force is newton (N).

Acceleration and Velocity

To understand acceleration, it is useful to understand what velocity is. Velocity is the rate of change in position (displacement) that occurs during a given time. The unit of velocity in the SI system is the metre per second (m/s).

Example 1

If a body moves on its x-axis from the 10 m position to the 30 m position in 4 seconds, then it will change position by 20 m in 4 seconds. What is its velocity?

Solution 1

$$\text{Velocity} = \frac{\text{Change in position (displacement)}}{\text{Time}}$$

$$= \frac{20\text{ m}}{4\text{ s}}$$

$$= \mathbf{5\ m/s\ (Ans.)}$$

Acceleration is the rate of change in velocity that occurs during a given time. The unit of acceleration in the SI system is the metre per second squared (m/s^2).

Note: Scalar quantities are described with a magnitude only. Vector quantities are described with both a magnitude and direction.

Scalar Speed: 20 kilometers per hour

Vector Velocity: 20 kilometers per hour, Northwest.

Example 2

A body is moving toward the right at a velocity of 10 m/s. After 4 seconds, it is moving to the right at a velocity of 30 m/s. It has changed velocity by 20 m/s (from 10 m/s to 30 m/s) during the 4 seconds. What is its rate of acceleration?

→	→
v = 10 m/s	v = 30 m/s
time = NOW	time = NOW + 4 seconds

Solution 2

$$\begin{aligned}\text{Acceleration} &= \frac{\text{Change in velocity}}{\text{Time}}\\ &= \frac{20\text{ m/s}}{4\text{ s}}\\ &= \frac{20}{4} \times \frac{\text{m/s}}{\text{s}}\\ &= \mathbf{5\ m/s^2\ (Ans.)}\end{aligned}$$

Note: A velocity of 5 m/s means that a body's position will change by 5 m over the duration of 1 second (a rate of change of position of 5 m/s).

An acceleration of 5 m/s^2 means that a body's velocity will change by 5 m/s over the duration of 1 second (a rate of change of velocity of 5 m/s per second, or 5 m/s^2).

Relationship between Mass, Force, and Acceleration

For all cases:

$$\text{Force} = \text{mass} \times \text{acceleration}$$

If the unit of mass in the SI system is the kg and the unit of acceleration is m/s^2, then:

$$\begin{aligned}\text{Force} &= \text{mass (kg)} \times \text{acceleration (m/s}^2\text{)}\\ &= \text{kgm/s}^2\end{aligned}$$

In the SI system the unit of force, kgm/s^2 is called a newton (N).

That is

$$1\text{ N} = 1\text{ kg} \times 1\text{ m/s}^2$$

This means

If a force of 1 N is applied to a mass of 1 kg, then that mass will accelerate at 1 m/s^2.

Or

If a mass of 1 kg is observed to be accelerating at 1 m/s^2, then there must be a force of 1 N acting upon the mass.

Or

If an unknown mass is found to be accelerating at 1 m/s^2 under the influence of a 1 N force, then that mass must be 1 kg.

Example 3

A mass of 1000 kg is to be accelerated at 15 m/s^2. What force is required?

Solution 3

$$
\begin{aligned}
\text{Force} &= \text{mass (kg)} \times \text{accleration } (m/s^2) \\
&= 1000 \text{ kg} \times 15 \text{ m/s}^2 \\
&= 15\,000 \text{ kgm/s}^2 \\
&= 15\,000 \text{ N} \\
&= \mathbf{15 \text{ kN (Ans.)}}
\end{aligned}
$$

Since the newton is a relatively small unit, the kilonewton (kN) is often used.

$$1 \text{ kN} = 1000 \text{ N}$$

Acceleration Due to Gravity

If a body is within the earth's field of gravity, there is an attraction to the earth's centre due to this gravitational "pull". A falling body will fall towards the earth at an increasing velocity due to the pull of gravity. The velocity will increase by approximately 9.81 m/s over the duration of each second of time. This principle is referred to as acceleration due to gravity.

$$
\begin{aligned}
\text{Acceleration due to gravity} &= \frac{\text{Change in velocity}}{\text{Time}} \\
&= \frac{9.81 \text{ m/s}}{1\text{s}} \\
&= 9.81 \text{ m/s}^2
\end{aligned}
$$

This number is called the "gravitational constant," and must be remembered because it is of great importance when dealing with mechanics problems.

Force of Gravity

Since the acceleration due to the earth's gravity may be taken as 9.81 m/s^2, then the force of the earth's gravity acting on a mass of 1 kg will be:

$$
\begin{aligned}
\text{Force} &= \text{mass (kg)} \times \text{acceleration } (m/s^2) \\
&= 1 \text{ kg} \times 9.81 \text{ m/s}^2 \\
&= 9.81 \text{ N}
\end{aligned}
$$

The direction of this force will be downward (i.e., toward the earth). A mass of 1 kg on the earth's surface will be subjected to a downward force (force due to pull of gravity) of 9.81 N.

Example 4

A body has a mass of 200 kg. What will be the magnitude of the force due to the earth's gravity acting upon this mass?

Solution 4

$$
\begin{aligned}
\text{Force} &= \text{mass (kg)} \times 9.81 \text{ (N/kg)} \\
&= 200 \text{ kg} \times 9.81 \text{ N/kg} \\
&= \mathbf{1962 \text{ N (Ans.)}}
\end{aligned}
$$

Weight

Weight: A Controversial Term

The mass of a body is unchanging and does not vary according to its location.

The force of gravity on a body varies from location to location, depending on the value of the acceleration due to gravity. The value, 9.81 m/s^2, is only an average value for the acceleration due to the earth's gravity on the surface of the earth. It is much less at a distance of 50 km above the earth's surface.

It is for this reason that the SI system expresses the quantity of material (e.g. meat or concrete) in terms of the kilogram (an unchanging mass unit) rather than the newton. The newton could be used as a unit of measurement of the imprecise concept "weight" (which is an inexact word often used to denote the force of gravity on the material). However, in the SI system, the term "weight" is not used.

Despite this, the term "weight" has sometimes been used to mean "mass" and sometimes to mean "force of gravity". In engineering courses, a clear distinction must be made between mass and force of gravity. Therefore, the **Canadian Standards Association** recommends that the term "weight" be avoided.

Self-Test 1

A body with a mass of 1000 kg moves horizontally from its original position A to point B, 24 m away in 3 seconds. The body is then accelerated for 2.5 s to point C. The new velocity at point C is 12 m/s.

a) Calculate the constant velocity between points A and B.

b) Calculate the acceleration from points B to C.

c) Calculate the force applied between points B and C.

8 m/s (Ans. a)

1.6 m/s^2 (Ans. b)

1600 N (Ans. c)

Objective 2

Perform simple calculations involving force, pressure, work, power, and energy.

Force and Pressure

Force

Force has previously been defined as the action on a body that tends to change its size, shape, state of rest, or state of motion. In the SI system, the unit of force is called the newton (N).

Pressure

Pressure is defined as force per unit area, and acts in a direction normal to (or at a right angle to) a surface.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

The unit of pressure in the SI system is defined as the pressure resulting from a force of 1 N acting uniformly over an area of 1 m^2. This unit is the pascal (Pa).

$$1\ \text{Pa} = 1\frac{\text{N}}{\text{m}^2}$$

The pascal is a very small unit; both kilopascals (1000 Pa) and megapascals (1 000 000 Pa) are often used when dealing with high pressures.

$$1\ \text{kPa} = 1000\ \frac{\text{N}}{\text{m}^2}$$

$$1\ \text{MPa} = 1\ 000\ 000\ \frac{\text{N}}{\text{m}^2}$$

Example 5

A force of 1000 N is exerted uniformly over an area of 0.25 m^2. What is the pressure?

Solution 5

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Force} = 1000\ \text{N}$$

$$\text{Area} = 0.25\ \text{m}^2$$

$$\text{Pressure} = \frac{1000\ \text{N}}{0.25\ \text{m}^2}$$

$$= 4000\frac{\text{N}}{\text{m}^2}$$

$$= \mathbf{4000\ Pa\ or\ 4\ kPa\ (Ans.)}$$

Work, Power and Energy

Work

If a force is applied to a body and causes it to move through a distance, then work is done. Work is the product of the force applied (newtons) and the distance moved (metres).

Work done = force (N) × distance (m)

Therefore, the work done when a force of one newton moves through a distance of 1 metre is one newton metre (Nm). This unit of work is called a joule (J).

1 joule (J) = 1 newton metre (Nm)

Example 6

A force of 100 N acting on a body moves it a distance of 10 m. What is the work done by the force on the body?

Solution 6

$$\begin{aligned} \text{Work done} &= \text{Force} \times \text{Distance} \\ &= 100\ \text{N} \times 10\ \text{m} \\ &= 1000\ \text{Nm} \\ &= 1000\ \text{J} \end{aligned}$$

Note: 1000 joules = 1 kilojoule

Work done = **1 kJ (Ans.)**

Power

Power is the rate of doing work; i.e., it is the quantity of work done in a given time.

$$\begin{aligned} \text{Power} &= \frac{\text{Work Done}}{\text{Time}} \\ &= \frac{\text{Nm}}{\text{s}} \\ &= \frac{\text{joule}}{\text{s}} \end{aligned}$$

The unit joule/s is a watt.

$$\frac{1\ \text{joule}}{\text{s}} = 1\ \text{watt}$$

The watt (W) is a small unit of power, so the units kilowatt or megawatt are used for many applications.

$$\begin{aligned} 1\ \text{kilowatt (kW)} &= 1000\ \text{W} \\ 1\ \text{megawatt (MW)} &= 1\,000\,000\ \text{W} \end{aligned}$$

Self-Test 2

If a force of 100 N acts on a body which moves a distance of 10 m in 5 seconds, what power is developed?

200 W (Ans.)

Energy

Energy is defined as the capacity of a body or substance to perform work. In other words, a body possesses energy when it is capable of doing work. Energy can be contained in many forms, and its presence is observed only by its effects.

Some forms of energy are solar, chemical, nuclear, electrical, and mechanical. The study of mechanics is concerned with the two forms of mechanical energy:

- Potential
- Kinetic

Potential Energy

Potential energy is the ability of a body to do work by virtue of its position. For example, water stored behind a dam contains potential energy due to its position and can be made to do work. If the water is released, it will flow to a lower point due to the force of gravity. A stretched spring also has potential energy, which increases as it is stretched (within certain limits).

Potential energy due to gravity can be expressed as:

$$\begin{aligned} \text{Potential Energy (PE)} &= \text{Mass} \times \text{Gravitational Force} \times \text{Vertical Height} \\ \text{PE} &= m \times g \times h \\ \text{Where } g &= \text{the force of gravity} = 9.81 \text{ N/kg} \\ h &= \text{vertical height, m} \end{aligned}$$

The units are $\left(\text{kg} \times \frac{\text{N}}{\text{kg}}\right) \times \text{m}$, or $\text{N} \times \text{m}$, or joules (J), which are the units of work.

Kinetic Energy

Kinetic energy (KE) is the ability of a body to do work due to its motion.

$$\text{Kinetic Energy (J)} = \frac{1}{2} \times \text{mass (kg)} \times \text{velocity}^2 \left(\frac{\text{m}}{\text{s}}\right)^2$$

The units are:

$$= \text{kg} \times \frac{\text{m}^2}{\text{s}^2}$$

$$= \frac{\text{kgm}}{\text{s}^2} \times \text{m}$$

$$= \text{N} \times \text{m}$$

$$= \text{Joule}$$

Chapter Summary

This chapter showed the importance of force. A force applied over an area creates pressure. A force used to move an object over a distance is said to do work. Energy is defined as the ability to do work. Energy may be solar, chemical nuclear, or mechanical. This chapter covered kinetic and potential energy, which are forms of mechanical energy.

Notice that all of the above come down to the idea of force. It is important to fully understand what a force is before further studies in Mechanics.

Once again, a force can be defined as any action that causes a body to change. If the net force acting on an object is greater than zero, the object will accelerate, according to Newton's Second Law. One important force that this chapter covered was the earth's gravitational force.

CHAPTER 2

Forces and Moments

Learning Outcome

When you complete this chapter you should be able to:

Perform calculations involving forces and moments, and determine when a system of forces is in equilibrium.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Define the moment of a force and its units.*
2. *Determine the direction and calculate the magnitude of the moment of a force.*

Chapter Introduction

When a force is applied to an object that is either stationary or in motion, that force will change the object's motion. Forces may also change a moving object's direction of travel.

The term "moment" means the product of a force times the distance from a centre point.

This chapter deals with the forces producing moments on beams that are fixed at some point. Additionally, if the beam was free to move, these forces would cause the beam to rotate around the fixed point.

Recall playing on a seesaw. A seesaw is an example of a mechanical lever and a fulcrum. A perfectly balanced seesaw will remain stationary about its rotating point (the tip of the fulcrum) until a force is applied at either end. This force, likely from the weight of a child, will cause rotation unless balanced by another force on the opposite end of the seesaw. The forces at either end of the seesaw create what are known as moments.

It is important to understand how forces and moments act on an object. A practical example would be choosing the right location and right number of supports for a pipe carrying steam.

This chapter will introduce the concepts of moments and how they are defined and calculated. The material will bring an understanding that moments create rotations. In order to keep an object at rest, one must balance not only the linear forces that act on that object but also the moments.

The study of **Scalars and Vectors** will provide the learner with additional information about forces that move objects in specific directions.

OBJECTIVE 1

Define the moment of a force and its units.

FORCE

A force is the pull or push exerted on a body; it may make a body move or bring it to rest.

For example, the pulling force of a locomotive on a string of boxcars causes the train to move; whereas the pushing force exerted by the brake pads on the rotating wheels causes it to stop.

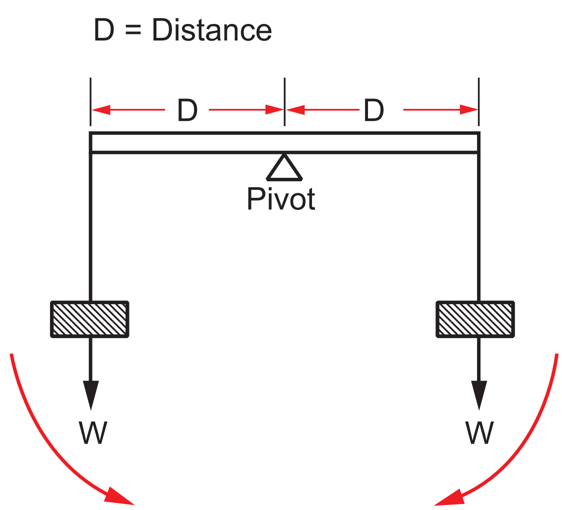
When several balanced forces act on a body, no change in movement will take place and the body and system of forces is said to be in equilibrium.

Figure 1 shows a simple pivot type balance with bodies of equal mass on each side. The system is in equilibrium and the cross arm remains horizontal. Each mass exerts a turning effect about the pivot. The mass on the right exerts a clockwise turning effect; the one on the left exerts a counterclockwise effect, as indicated by the two arrows on the bottom of Figure 1. The magnitude of this twisting or turning effect depends upon the:

a) size of the masses

b) distance of the suspension points from the pivot or fulcrum

Figure 1 – Simple Pivot Type Balance

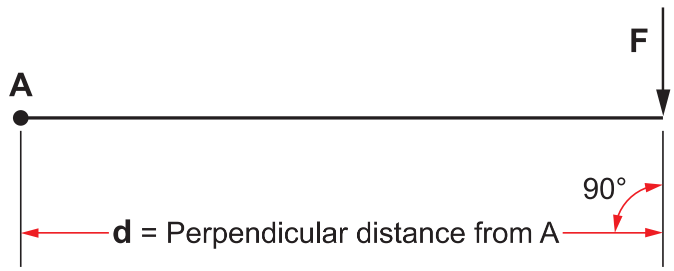


Moment of Force

A force, acting on a body at any distance from a point on that body, will tend to produce rotation around that point. The moment or turning moment of a force around a point is equal to the force multiplied by the perpendicular distance from the line of force to the point.

In Figure 2, the moment of force F about point A is F × d. Force is measured in newtons (or kN) and the distance in metres. Therefore, the moment of a force will be measured in newton metres (Nm) or kilonewton metres (kNm). The moment has a rotational direction also applied to it, expressed as clockwise or counterclockwise.

Figure 2 – Moment of Force



Example 1

In Figure 2, a force of 300 N acts at a perpendicular distance of 2 m from point A. What turning moment will be produced? What is its direction?

Solution 1

The force will produce a clockwise rotation.

Moment = Force × Perpendicular distance

= 300 N × 2 m

= **600 Nm clockwise (Ans.)**

Self-Test 1

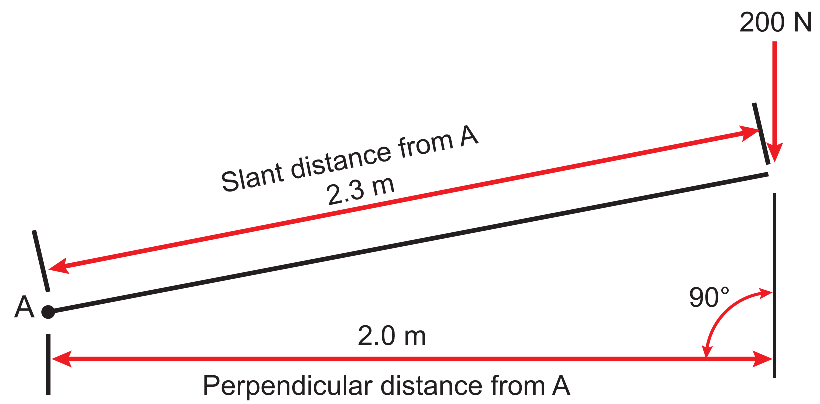
In Figure 2, a force of 450 N acts at a perpendicular distance of 1.75 m from point A. What turning moment will be produced? What is its direction?

787.5 Nm Clockwise (Ans.)

Example 2

A force of 200 N acts in the direction shown in Figure 3. What would be the turning moment of this force about point A?

Figure 3 – Directional Force



Solution 2

Moment = Force × Perpendicular distance

= 200 N × 2 m

= **400 Nm (Ans.)**

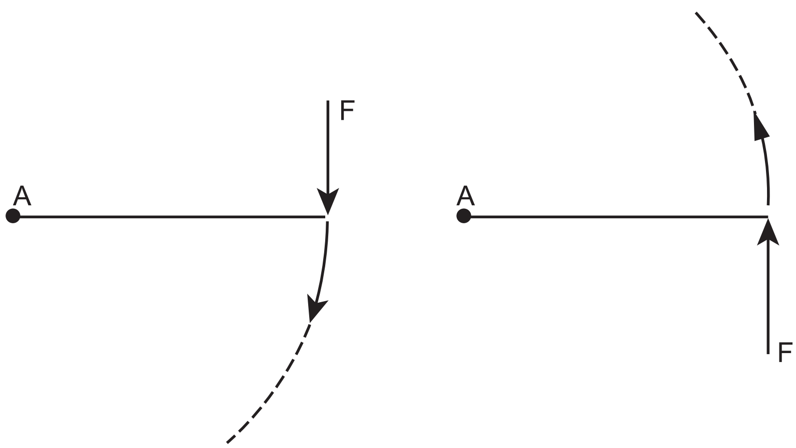
Objective 2

Determine the direction and calculate the magnitude of the moment of a force.

Direction of a Moment of Force

A force, acting at a distance from a point, will produce or tend to produce rotation about a point, with the point as centre. This rotation will either be in a clockwise or counterclockwise direction, as shown in Figure 4. This direction can be determined by imagining what would happen if the force were free to rotate about the point.

Figure 4 – Moment Directions



The bar shown in Figure 5 is pivoted at point P. The only forces applied are A and B at point E, which are equal in magnitude, but opposite in direction of action. Under this form of loading, the bar will be in equilibrium.

Figure 5 – Rotational Equilibrium – Equal Forces

Fulcrum

A single support about which a bar is free to rotate is called a fulcrum. In Figure 8 of Example 3, the support at A would be the fulcrum.

Equilibrium

If the forces, and the moments of force, are such that there is no movement or rotation of the bar, the system is said to be in equilibrium.

The conditions for equilibrium are:

upward forces = downward forces

forces acting to the right = forces acting to the left

clockwise moments = counterclockwise moments

In Figure 6, the forces A and B are equal in magnitude and opposite in direction of action. The forces are acting at different distances from the pivot point with d_2 greater than d_1. The moments that the forces are producing are not the same. The moment of force A about the pivot point is $d_1 \times A$ which will be less than the moment of force B about the pivot point which is $d_2 \times B$. Therefore, the bar is not in equilibrium and will rotate in a counterclockwise direction.

Figure 6 – Rotational Equilibrium – Equal Forces but Unequal Moments

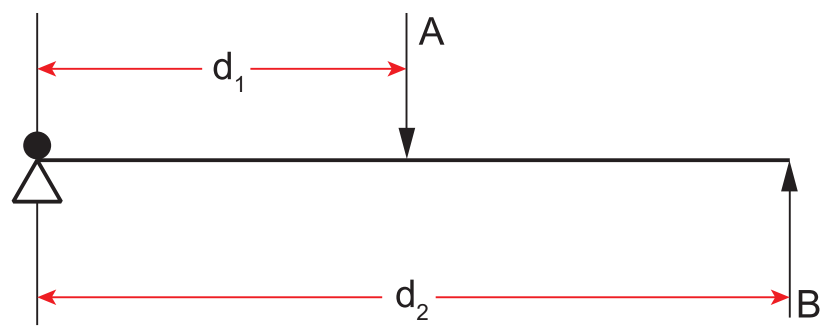
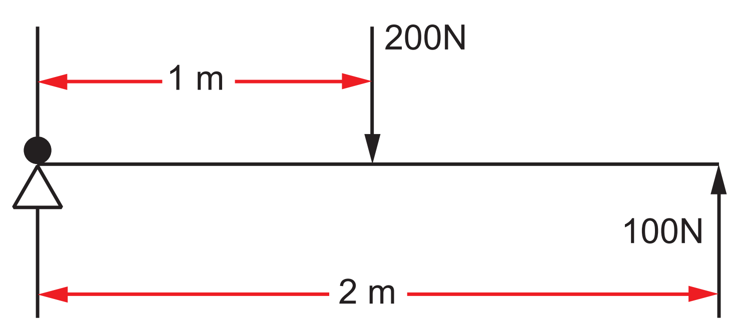


Figure 7 shows a bar with the same dimensions as that in Figure 6, but the values of forces A and B have been changed. The moments of the forces taken about the pivot are equal, so the bar is in equilibrium.

1 m × 200 N = 200 Nm clockwise moment, due to force A

2 m × 100 N = 200 Nm counterclockwise moment, due to force B

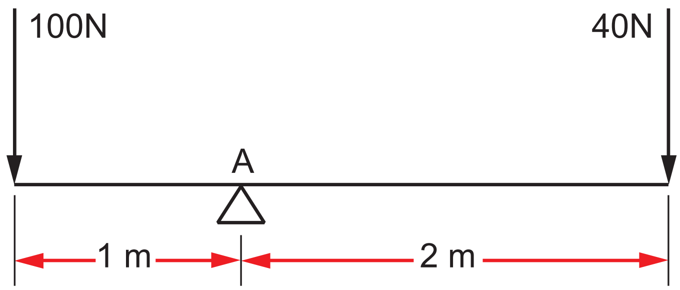
Figure 7 – Rotational Equilibrium – Unequal Forces but Equal Moments



Example 3

A bar of negligible mass is supported at A and loaded, as shown in Figure 8. What are the clockwise and counterclockwise moments of force at the pivot point, and in which direction will the bar rotate?

Figure 8 – Supported Bar



Solution 3

Taking moments about A:

Clockwise moment = 40 N × 2 m

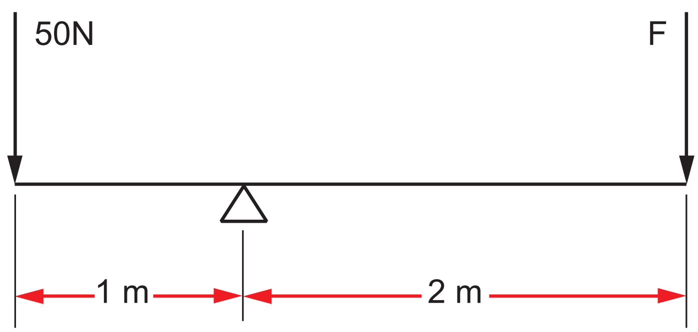
= **80 Nm (Ans.)**

Counterclockwise moment = 100 N × 1 m

= **100 Nm (Ans.)**

Since the counterclockwise moment is greater than the clockwise moment, **the bar would rotate in a counterclockwise direction (Ans.)**.

Figure 9 – Negligible Mass Bar



Self-Test 2

A bar of negligible mass supports a load as shown in Figure 9. What force F is required at the right hand end of the bar for equilibrium?

25 N (Ans.)

Note: In solving moment problems, always:

a) state that moments are being taken about the fulcrum

b) state that clockwise moments = counterclockwise moments

c) substitute known values and solve for the unknown value

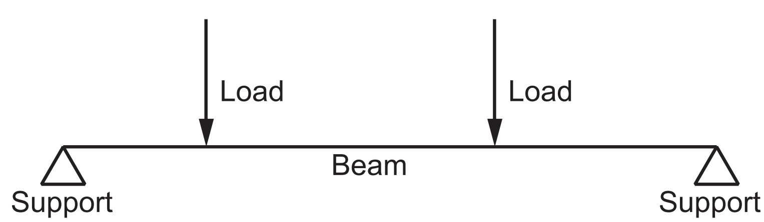
BEAMS

A beam may be defined as a rigid member or bar, supported in some way so that it is capable of carrying a load or system of loads.

Simply Supported Beams

If the beam rests on supports so that it is free to bend without restriction from the supports, it is called simply supported (Figure 10).

Figure 10 – Simply Supported Beam



Point Loads

If the loads on the beam can be considered to be concentrated at specific points, they are called point loads.

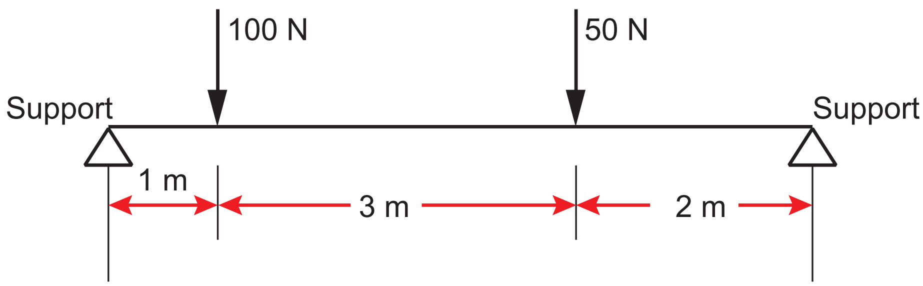
Reaction at Supports

For equilibrium, the downward forces on the beam must be resisted by reaction forces at the supports.

Example 4

A 6 m beam is simply supported at each end and carries point loads (Figure 11). Calculate the reactions at each support to achieve equilibrium of the beam.

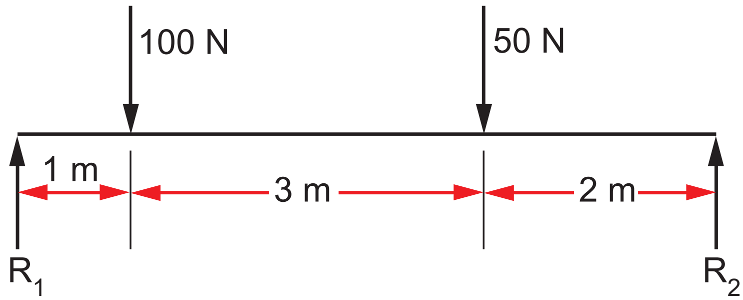
Figure 11 – Simply Supported Beam



Solution 4

The end reactions may be considered as upward forces to resist the downward forces; it is normal practice to draw the free body diagram (Figure 12), where R_1 = reaction at left and R_2 = reaction at right.

Figure 12 – Free Body Diagram of Beam



To find R_1 and R_2:

Take moments about either R_1 or R_2. The moment of a force is the product of the force and its perpendicular distance from the point about which moments are taken. R_1 or R_2 can be selected as the point about which moments are taken, allowing the other reaction to be calculated.

First, take moments about R_1. This will allow for the calculation of R_2.

The 100 N force is 1 m from R_1, so its moment about R_1 is 100 N × 1 m = 100 Nm. If the beam were free to move about R_1, the 100 Nm moment would cause a clockwise rotation about R_1.

The 50 N force is 4 m from R_1, so its moment about R_1 is 50 N × 4 m = 200 Nm. It would also cause clockwise rotation about R_1.

The reaction force R_2 is 6 m from R_1, so that its moment about R_1 is R_2 × 6 m = 6 R_2 Nm. This moment would tend to cause a counterclockwise rotation of the beam about R_1.

For equilibrium:

$$\begin{aligned}\text{Clockwise moments} &= \text{Counterclockwise moments}\\ 100\text{ Nm} + 200\text{ Nm} &= 6\text{ m} \times R_2\\ 300\text{ Nm} &= 6\text{ m} \times R_2\\ R_2 &= \frac{300\text{ Nm}}{6\text{ m}}\\ &= \mathbf{50\ N\ (Ans.)}\end{aligned}$$

R_1 can now be found either by equating upward and downward forces or by taking moments about R_2.

For equilibrium:

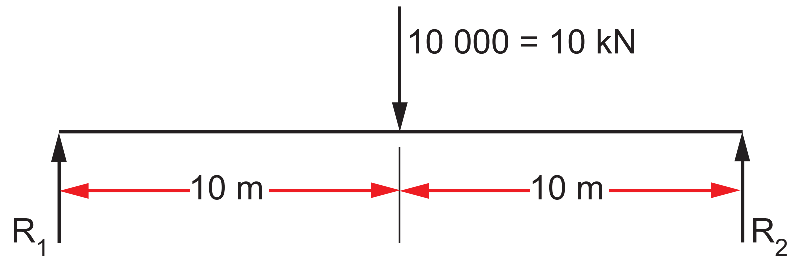
$$\begin{aligned}\text{Upward forces} &= \text{Downward forces}\\ R_1 + R_2 &= 100\text{ N} + 50\text{ N}\\ R_1 + 50\text{ N} &= 150\text{ N}\\ R_1 &= 150\text{ N} - 50\text{ N}\\ &= \mathbf{100\ N\ (Ans.)}\end{aligned}$$

Check:

Taking moments about R_2:

$$\begin{aligned}\text{Clockwise moments} &= \text{Counterclockwise moments}\\ R_1 \times 6\text{ m} &= (100\text{ N} \times 5\text{ m}) + (50\text{ N} \times 2\text{ m})\\ R_1 \times 6\text{ m} &= 500\text{ Nm} + 100\text{ Nm}\\ R_1 \times 6\text{ m} &= 600\text{ Nm}\\ R_1 &= \frac{600\text{ Nm}}{6\text{ m}}\\ &= \mathbf{100\ N\ (Ans.)}\end{aligned}$$

Figure 13 – Simply Supported Beam for Self-Test 3



Self-Test 3

A beam 20 m long (Figure 13) is simply supported at each end and carries a load of 10 000 N at the centre of the span. Calculate each reaction by taking moments.

$R_1 = R_2 = 5$ kN (Ans.)

CHAPTER SUMMARY

A moment (Nm) about a point of an object is created when a force acts on that object at a certain perpendicular distance away from that point.

In order for an object to be at equilibrium, balance the vertical and horizontal forces, as well as all the moments. In other words, all upward forces must equal all the downward forces, all the forces acting to the left must balance those to the right, and counterclockwise moments must balance clockwise moments.

When solving a problem involving moments and forces, choose any point of rotation to calculate the moment. However, choose a point that has an unknown reaction force acting at that point. This will simplify calculations.

It is also important to visualize how a force acts around the rotation point and whether it creates a counterclockwise or clockwise rotation. A perfect example of an object designed to support loads is a beam.

As was shown in this chapter, beams require supports. The reaction forces at these points must be calculated, given that a certain load is applied. Therefore, when solving problems involving beams it is easiest to calculate the moments about a supporting point.

CHAPTER 3

Simple Machines

LEARNING OUTCOME

When you complete this chapter you should be able to:

Perform calculations relating to mechanical advantage, velocity ratio and efficiency.

LEARNING OBJECTIVES

Here is what you should be able to do when you complete each objective:

1. *Define the term simple machine and apply to calculations of mechanical advantage, velocity ratio and efficiency of simple machines.*

Chapter Introduction

Often, machines are required to help with tasks having loads too great for humans to bear.

This chapter introduces the concept of a simple machine and the benefits of simple machines when used correctly. An example of a simple machine is the fulcrum and lever.

Recall previously studying moments and forces. Now consider a seesaw, 1 m in length, where the fulcrum is not in the center, but positioned one quarter of the way in from the left hand side. Imagine a load of 100 N applied on the left hand side of the seesaw. In order to raise this load, and balance the moment, a smaller load of only 33 N would have to be applied on the right hand side. By using a simple machine, a large load was raised by applying a small force.

This chapter covers simple machines and shows how to calculate their mechanical advantage. The concepts of velocity ratio and efficiency are also introduced. Note that no machine in the real world is perfect. Because of imperfections that cause friction, a machine will never be 100% efficient.

Simple machines are a part of everyday life and often go unnoticed. A bottle opener is a wonderful example of a lever.

Objective 1

Define the term simple machine and apply to calculations of mechanical advantage, velocity ratio and efficiency of simple machines.

Simple Machines

A machine may be defined as a device that receives energy from some source and uses that energy to do work. A simple machine is one that receives energy by means of a single applied force, and produces work by means of a single output force.

In all machines, the work output is always less than the work input. This is because the machine must work to overcome internal friction and other resistive forces.

Lifting machines, in particular, are often arranged so that a relatively small effort can raise a relatively large load. The machine has a mechanical advantage.

Actual Mechanical Advantage (MA)

In any machine, the ratio of the load to the effort is called the actual mechanical advantage of the machine.

$$\text{Actual mechanical advantage (MA)} = \frac{\text{Load}}{\text{Effort}}$$

Since the load and effort have the same units, MA is simply a number without units, which indicates the advantage of using the machine.

Example 1

A lever is used to move a load of 1000 N by applying an effort of 100 N. What is the mechanical advantage of the lever?

Solution 1

$$\begin{aligned}\text{Actual MA} &= \frac{\text{Load}}{\text{Effort}} \\ &= \frac{1000\text{ N}}{100\text{ N}} \\ &= \mathbf{10\ (Ans.)}\end{aligned}$$

That is, this lever gives an advantage of 10:1.

Levers

A lever is a straight bar or other rigid structure, supported at a fulcrum in such a way that a small force (or effort) can balance or move a much larger load.

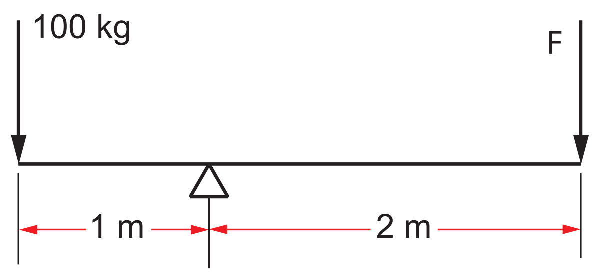
Forces at Fulcrum (Reaction)

In a horizontal lever system, there must be a force or reaction at the fulcrum, to oppose and balance the other forces (upward and downward) on the system.

Example 2

Find the force necessary to just move the 100 kg mass on the end of the lever shown in Figure 1. Find the magnitude and direction of the reaction at the fulcrum. Find the mechanical advantage of the system. Neglect the mass of the lever itself.

Figure 1 – Lever



Solution 2

Note: The mass of 100 kg must be converted to a load in newtons:

$$100 \text{ kg} \times \frac{9.81 \text{ N}}{\text{kg}} = 981 \text{ N}$$

Taking moments about the fulcrum:

$$\text{Clockwise moments} = \text{Counterclockwise moments}$$

$$F \times 2 \text{ m} = 981 \text{ N} \times 1 \text{ m}$$

$$F = \frac{981 \text{ Nm}}{2\text{m}}$$

$$= \textbf{490.5 N acting downward on the right (Ans.)}$$

$$\text{Upward forces} = \text{Downward forces}$$

$$\text{Force at fulcrum} = 490.5 \text{ N} + 981 \text{ N}$$

$$= \textbf{1471.5 N upwards (Ans.)}$$

$$\text{Actual MA} = \frac{\text{Load}}{\text{Effort}}$$

$$= \frac{981 \text{ N}}{490.5 \text{ N}}$$

$$= \textbf{2 (Ans.)}$$

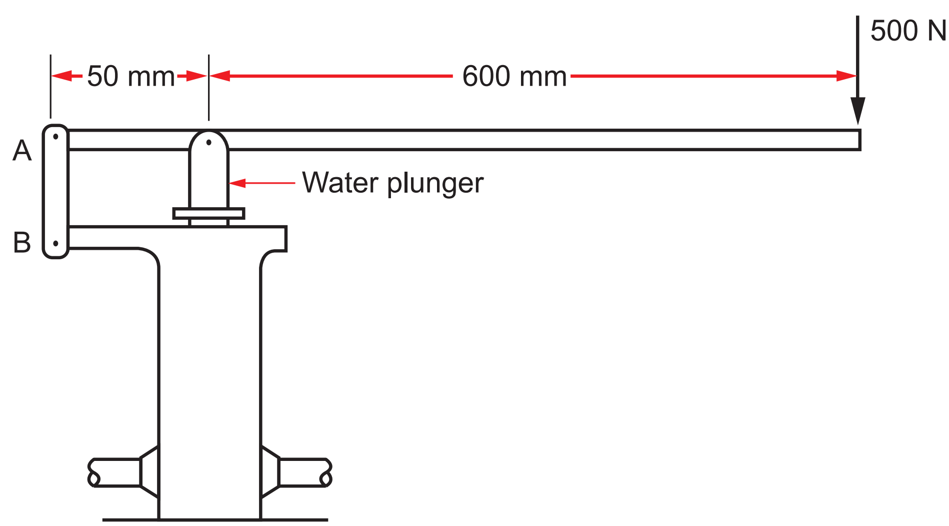
Self-Test 1

A small hand pump is shown in Figure 2. An effort of 500 N applied to the right end of the lever is required to overcome the resistance of the water plunger. Calculate the force exerted by the plunger on the lever and the mechanical advantage of the lever.

Force exerted by the plunger = 6500 N upward (Ans. a)

MA = 13 (Ans. b)

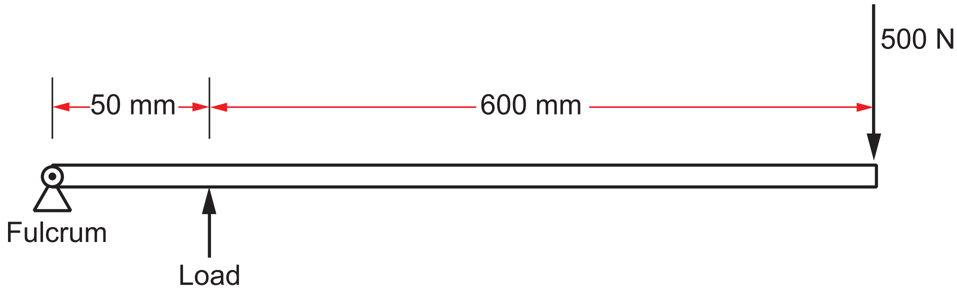
Figure 2 – Small Hand Pump



Hint

The system may be simplified to a lever system, as shown in Figure 3, with the fulcrum at the left hand end.

Figure 3 – Lever System



Velocity Ratio (VR)

The velocity ratio of a machine is the ratio of the distance moved by the effort to the distance moved by the load.

$$\text{Velocity ratio (VR)} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

Since the same units appear on the top and bottom of the equation, VR is a number without units.

Example 3

In a lifting machine, the effort applied moves a distance of 1 m, while the load moves 100 mm. What is the velocity ratio?

Solution 3

$$\text{VR} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{1\ \text{m}}{0.1\ \text{m}}$$

$$= \mathbf{10\ (Ans.)}$$

Therefore, the effort moves 10 times further than the load.

Note: The units must be the same when determining velocity ratio.

Efficiency

The efficiency of a machine is determined by the ratio of the output work to the input work.

$$\text{Efficiency} = \frac{\text{Output work}}{\text{Input work}}$$

$$\text{Output work} = \text{Load} \times \text{Distance moved by load}$$

$$\text{Input work} = \text{Effort} \times \text{Distance moved by effort}$$

Therefore

$$\begin{aligned}
\text{Efficiency} &= \frac{\text{Load} \times \text{Distance moved by load}}{\text{Effort} \times \text{Distance moved by effort}} \\
&= \frac{\text{Load}}{\text{Effort}} \times \frac{\text{Distance moved by load}}{\text{Distance moved by effort}} \\
&= \text{Actual MA} \times \frac{1}{\text{VR}} \\
&= \frac{\text{Actual MA}}{\text{VR}}
\end{aligned}$$

$$\text{Percentage efficiency} = \frac{\text{Actual MA}}{\text{VR}} \times 100$$

Ideal Machine

If a machine had no losses, then the efficiency would be 100% (MA = VR). This would be the ideal mechanical advantage and would indicate a perfect machine.

Ideal MA = VR

Example 4

A machine is used to lift a load of 1000 N through a distance of one metre. If the effort applied is 100 N, what is the MA? What distance will the effort move if the efficiency of the machine is 75%?

Solution 4

$$\text{Load} = 1000\text{ N}$$

$$\text{Distance moved by the load} = 1\text{ m}$$

$$\text{Effort} = 100\text{ N}$$

$$\text{Efficiency} = 75\%\ (0.75)$$

$$\text{Actual MA} = \frac{\text{Load}}{\text{Effort}}$$

$$= \frac{1000\text{ N}}{100\text{ N}}$$

$$= \mathbf{10\ (Ans.)}$$

$$\text{Efficiency} = \frac{\text{Actual MA}}{\text{VR}}$$

$$\text{VR} = \frac{\text{Actual MA}}{\text{Efficiency}}$$

$$= \frac{10}{0.75}$$

$$= 13.33$$

$$= \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\text{Distance moved by effort} = \text{Distance moved by load} \times \text{VR}$$

$$= 1\text{ m} \times 13.33$$

$$= \mathbf{13.33\text{ m}\ (Ans.)}$$

Self-Test 2

An effort of 200 N is required to raise a mass of 200 kg in a certain machine. The mass is raised one metre, while the effort moves 10 m. Calculate the following:

a) VR

b) Actual MA

c) Efficiency

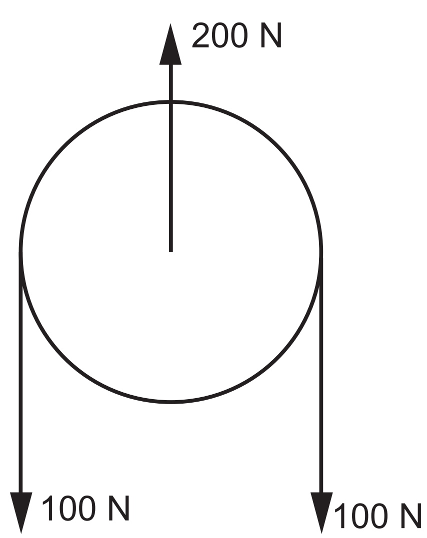
VR = 10 (Ans. a)
AMA = 9.81 (Ans. b)
Efficiency = 98.1% (Ans. c)

PULLEY SYSTEMS

Pulley systems are comprised of pulleys or wheels with ropes or chains. They are lifting machines designed to lift heavy loads by applying a relatively small effort.

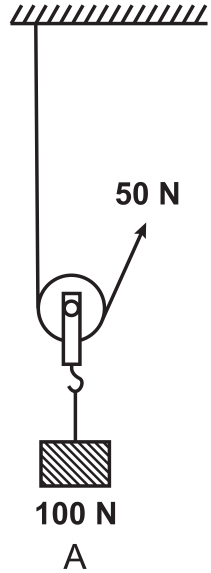
The simplest arrangement is a single pulley or wheel with a single rope (Figure 4). Although this system has no actual mechanical advantage (MA = 1), it is often used because it allows a load to be lifted vertically by a downward effort. In the single pulley system, using Figure 4 as an example, the force on the rope that supports the pulley (200 N) is equal to the sum of the load (100 N) plus the effort (100 N). Upward forces = Downward forces.

Figure 4 – Single Pulley



Another arrangement of a single pulley or wheel with a single rope is shown in Figure 5. This system has a mechanical advantage (MA) of 2. It is used to lift a load vertically, by an upward effort. In the single pulley system, using Figure 5 as an example, the load (100 N) is equal to the force on the rope that supports the pulley (50 N) plus the effort (50 N).

Figure 5 – Single Pulley



Block and Tackle System

Figure 6 shows a block and tackle system. It consists of top and bottom pulley blocks, each carrying a number of pulleys that rotate on a common axle. There may be an equal number of pulleys in each block, or one more pulley in one than in the other. The rope passes over each pulley in turn from top to bottom. One end of the rope is fastened to the block opposite the last pulley, and the other end is free to apply the effort. Both top and bottom blocks have hooks. The top hook supports the system of pulleys, and the bottom hook supports the load.

$$MA = \frac{\text{Load}}{\text{Effort}}$$

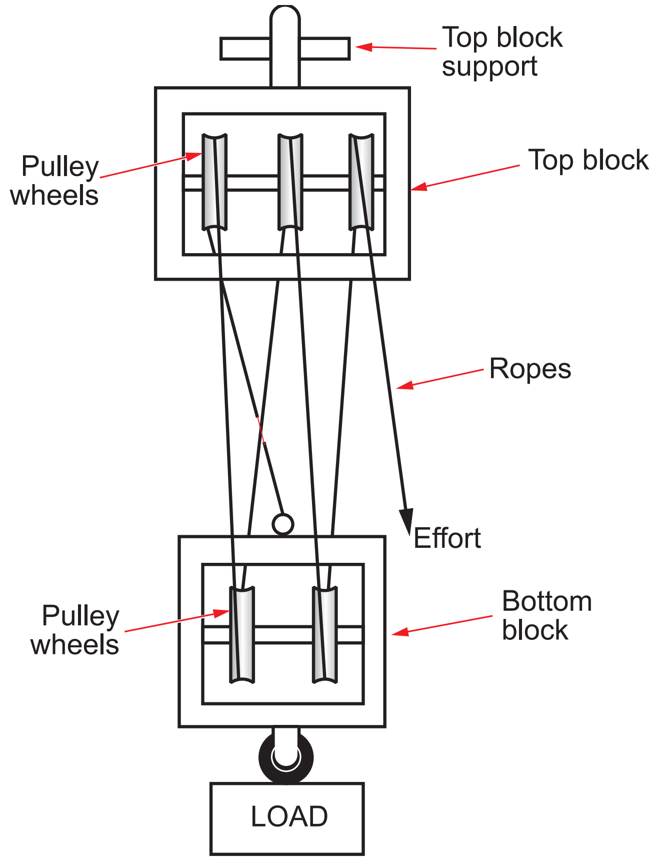
$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

As before, if losses are neglected:

$$VR = \text{Ideal MA}$$

The velocity ratio of a pulley block system is equal to the number of ropes supporting the load block. If the blocks are used, with the effort pulling downward, to lift a load vertically, the number of ropes supporting the bottom block is equal to the total number of pulleys in the system. In Figure 6, there are 5 pulleys; therefore, VR = 5.

Figure 6 – Block and Tackle

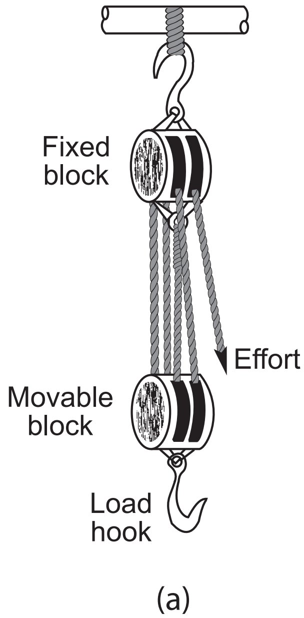


Downward Effort versus Upward Effort

Figure 7 demonstrates alternative arrangements for a pulley system. The same block and tackle, with four pulleys, is used in both Figure 7(a) and 7(b). However, in 7(b) the system is inverted, so that the effort must be directed upwards. This upward arrangement is very common, particularly in large cranes where the effort motor is located at the top of the crane.

The significant difference in operation between these two arrangements lies in the number of cable sections that are actually supporting (applying upward lift to) the load. Notice that, in Figure 7(a), there are four cable sections supporting the load. The effort is acting downwards from the top block, so it is not supplying any upward force to the load. However, in Figure 7(b), the effort cable is applying upward force to the load block, so the total number of cable sections supporting the load becomes five.

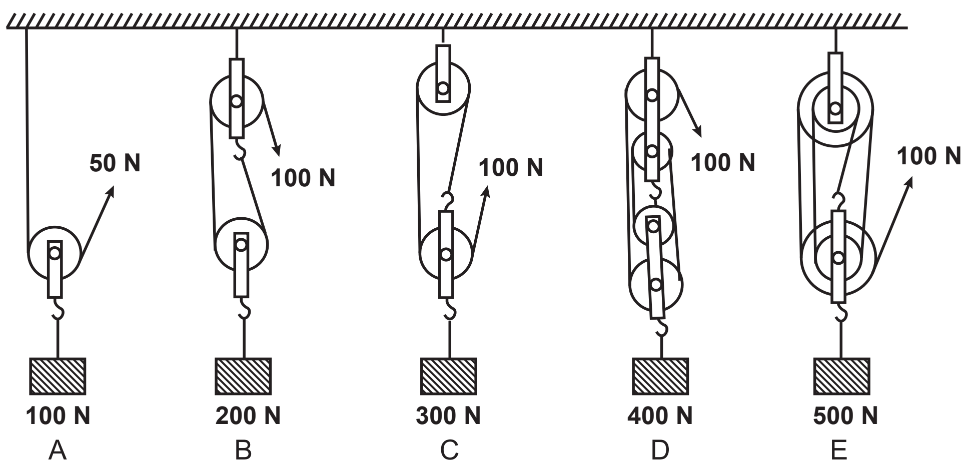
Figure 7 – Block and Tackle Systems



The significance of the above two arrangements becomes important when determining the velocity ratio for a pulley system.

Figure 8 shows some common block and tackle arrangements and the lifting force required for the given load forces, assuming 100% efficiency.

Figure 8 – Block and Tackle Systems



Example 5

A block and tackle system has 3 pulleys in each block. An effort of 100 N is required to raise a load of 480 N. Calculate the efficiency of the system (normal hookup with effort pulling down).

Solution 5

Total number of pulleys = 6

VR = 6 (normal hookup, pulling down)

$$\begin{aligned} \text{MA} &= \frac{\text{Load}}{\text{Effort}} \\ &= \frac{480\text{ N}}{100\text{ N}} \\ &= 4.8 \end{aligned}$$

$$\begin{aligned} \text{Efficiency} &= \frac{\text{MA}}{\text{VR}} \\ &= \frac{4.8}{6} \\ &= \mathbf{0.8\ (Ans.)} \end{aligned}$$

$$\begin{aligned} \%\ \text{Efficiency} &= 0.8 \times 100 \\ &= \mathbf{80\%\ (Ans.)} \end{aligned}$$

Self-Test 3

A block and tackle system has 4 pulleys in the upper block and 3 in the lower block. An effort of 250 N is required to raise a load of 1485 N. Calculate the efficiency of the system (normal hookup with effort pulling downward).

84.9% (Ans.)

Simple Wheel and Axle

A simple wheel and axle is shown in Figure 9. The load is applied to the smaller diameter axle and the effort to the wheel. The wheel and axle ropes are wound in opposite directions.

If the radius of wheel = R and diameter = D

Radius of axle = r and diameter = d

Taking moments about the centre of the axle, and neglecting losses:

$$\text{Clockwise moments} = \text{Counterclockwise moments}$$

$$\text{Load} \times r = \text{Effort} \times R$$

$$\text{from which,} \quad \text{Load} = \frac{\text{Effort} \times R}{r}$$

$$\text{and} \quad \frac{\text{Load}}{\text{Effort}} = \frac{R}{r}$$

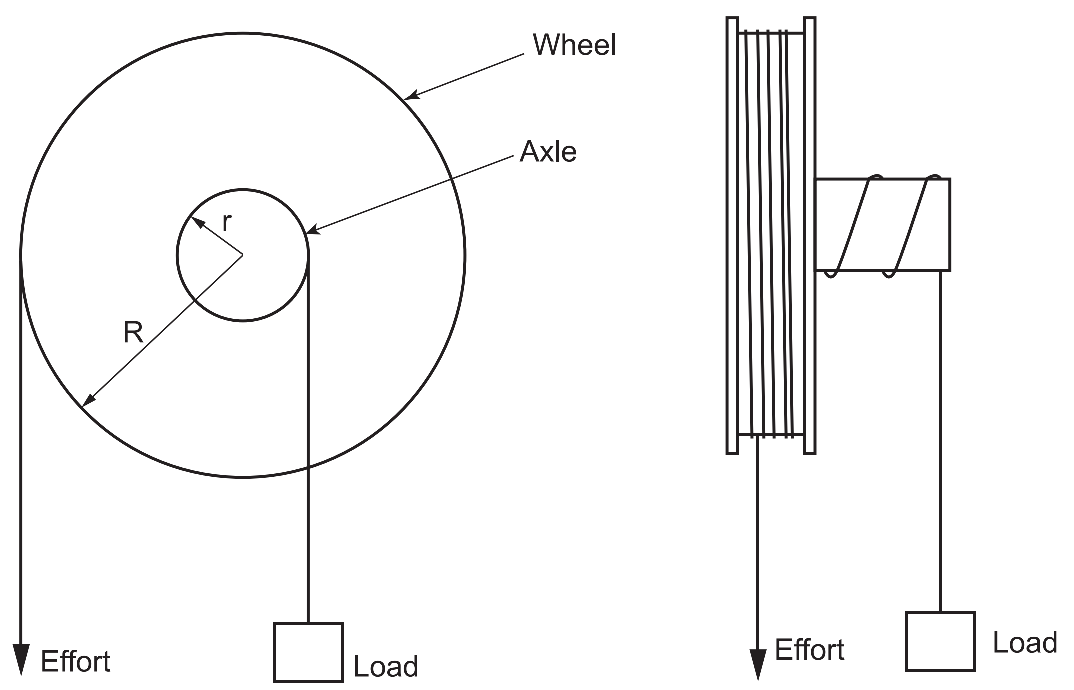
$$\text{Also,} \quad \text{Actual Mechanical Advantage, MA} = \frac{\text{Actual load}}{\text{Actual effort}}$$

$$\text{Velocity Ratio, VR} = \frac{\text{effort distance}}{\text{load}} = \frac{R}{r} = \frac{D}{d}$$

If there are losses, efficiency is less than 100%:

$$\%\ \text{Efficiency} = \frac{\text{MA}}{\text{VR}} \times 100$$

Figure 9 – Wheel and Axle



Example 6

A simple wheel and axle has a wheel diameter of 250 mm and axle diameter of 50 mm. Calculate the following:

a) VR
b) MA if the efficiency is 90%
c) Effort to raise a load of 500 N

Solution 6

a)
$$\begin{aligned} VR &= \frac{D}{d} \\ &= \frac{250\text{ mm}}{50\text{ mm}} \\ &= \mathbf{5\ (Ans.)} \end{aligned}$$

b)
$$\begin{aligned} \%\text{ Efficiency} &= \frac{MA}{VR} \times 100 \\ 90 &= \frac{MA}{5} \times 100 \\ MA &= \frac{90}{100} \times 5 \\ &= \mathbf{4.5\ (Ans.)} \end{aligned}$$

c)
$$\begin{aligned} MA &= \frac{\text{Load}}{\text{Effort}} \\ \text{Effort} &= \frac{\text{Load}}{MA} \\ &= \frac{500\text{ N}}{4.5} \\ &= \mathbf{111.11\ N\ (Ans.)} \end{aligned}$$

Self-Test 4

A simple wheel and axle has a wheel diameter of 300 mm and axle diameter of 75 mm. Calculate the following:

a) VR

b) MA if the efficiency is 87%

c) Effort to raise a load of 450 N

VR = 4 (Ans. a)

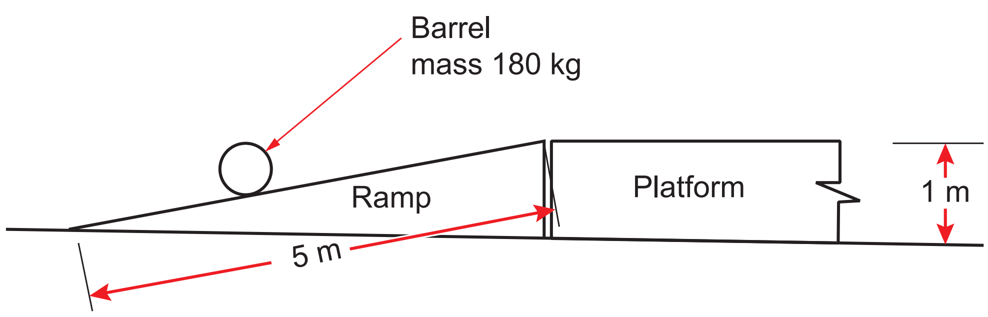
MA if the efficiency is 87% = 3.48 (Ans. b)

Effort to raise a load of 450 N = 129.31 N (Ans. c)

Inclined Plane

An inclined plane is a simple machine. The example in Figure 10 is a 5 m long ramp, which is used to raise a barrel of oil having a mass of 180 kg, and a vertical distance of 1 m.

Figure 10 – Inclined Plane



When the barrel of oil has been rolled up the ramp to the platform level, it will have a potential energy of:

180 kg × 9.81 N/kg × 1 m = 1765.8 Nm or 1765.8 J

If there is no friction, then the work done by rolling the barrel up the ramp will be equal to the output work, which is 1765.8 Nm. If there is friction, then some of the work done by rolling the barrel up the ramp will be turned into heat energy along the ramp. In this case, the input work will be greater than 1765.8 Nm.

The law of conservation of energy can be applied to simple machines and is written as:

Input work = Output work + Wasted work

Work in overcoming friction can be considered as wasted work.

Other simple machines that use the inclined plane principle are the screw jack and the wedge.

The screw jack is a special type of inclined plane. It is an inclined plane that has been wrapped around a cylinder. This simple machine is frequently used as part of a car's tool kit. The wedge is used to raise a heavy load a small distance, in order to insert some other device under it or to promote or restrain movement.

Chapter Summary

A simple machine makes work easier. By inputting a small force, using the mechanical advantage (MA) provided by the machine, it is possible to move a large load. Three key concepts of a machine include its mechanical advantage (MA), its velocity ratio (VR), and its efficiency.

MA = Load/Effort

VR = Distance moved by Effort/ Distance moved by Load

Efficiency = MA/VR = Work from Load / Work from Effort

An ideal machine is one that has the same mechanic advantage (MA) as its velocity ratio (VR). This is otherwise known as being 100% efficient.

Three types of simple machines were covered in this chapter: pulley systems, simple wheel and axel, and inclined planes.

CHAPTER 4

Scalars and Vectors

LEARNING OUTCOME

When you complete this chapter you should be able to:

Define and identify scalar and vector quantities and solve simple vector problems graphically.

LEARNING OBJECTIVES

Here is what you should be able to do when you complete each objective:

1. *Define scalar and vector quantities as they apply to drawing vector diagrams.*

Chapter Introduction

Often times in the English language words such as speed and velocity are used interchangeably. However, in Engineering these are two very different terms.

The statement that a car is moving at a speed of 20 m/s is very different from the statement that a car is moving at a velocity of 20 m/s North-West. Notice that speed only pertains to the magnitude of how quickly the vehicle is moving, whereas the velocity also gives us the direction in which it is moving. The difference between speed and velocity is that one is a scalar and one is a vector.

This chapter introduces the concept of scalar and vector quantities, and shows how they differ. It also goes into detail about the different types of vectors, and shows the importance of direction. Vector diagrams are featured and used to show how multiple vectors can be combined into one resulting vector.

Objective 1

Define scalar and vector quantities as they apply to drawing vector diagrams.

Scalar Quantities

A scalar quantity is one that has magnitude only and can be completely described by a number with the necessary unit. Quantities such as length, area, volume, time, and mass are scalars. For example, 3 metres describes a scalar quantity of length; 5 minutes a scalar quantity of time; and 4 cubic centimeters a scalar quantity of volume.

Vector Quantities

A vector quantity is a quantity that has magnitude and direction.

a) Magnitude - the numerical size of the quantity must be specified.

b) Direction - the direction of the action must be specified to describe a vector and can be stated in degrees, points of the compass, right, left, up, or down.

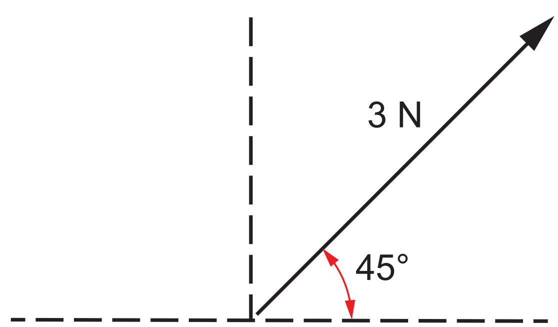
Vectors are often represented by arrows. The length of the line is a scale representation of the magnitude. The direction of the arrow represents the direction of action. Some examples of vector quantities are:

- Force
- Velocity
- Displacement

Force

A force of 3 newtons (magnitude), acting at 45° to the horizontal (direction) can be represented as shown in Figure 1.

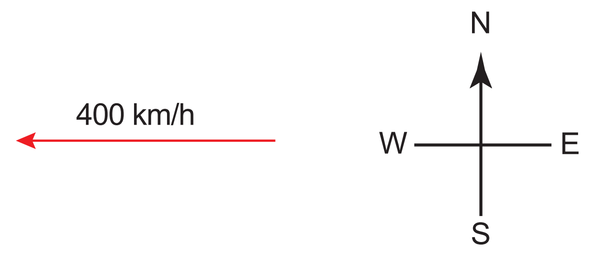
Figure 1 – Vector Representation of a Force



Velocity

An aircraft flying west (direction) at a speed of 400 kilometers per hour (magnitude) could be represented by a vector, as shown in Figure 2.

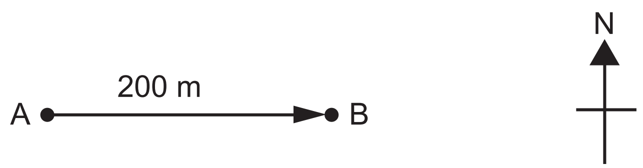
Figure 2 – Vector Representation of a Velocity



Displacement

If a man walks east (direction) a distance of 200 m (magnitude) from point A to point B, this act could be represented by a displacement vector, as shown in Figure 3.

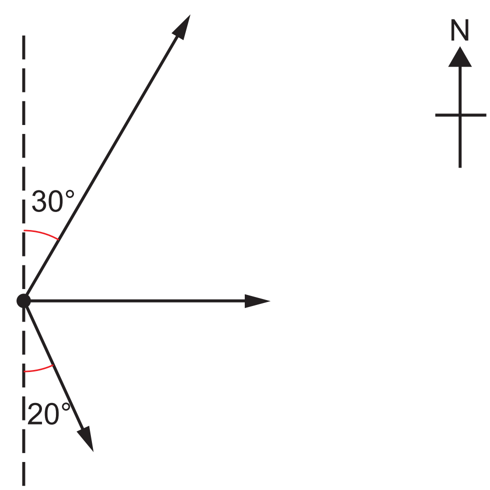
Figure 3 – Displacement Vector



Space Diagrams

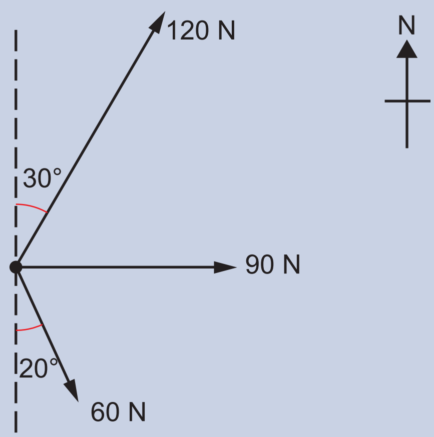
A space diagram illustrates a system of vectors. To draw a space diagram, a convenient scale is chosen so the diagram is a suitable size. The angles must be accurately drawn. In problems where the compass points are used to specify direction, the diagram should always state which direction is north. Figure 4 illustrates a common method.

Figure 4 – Space Diagram



Self-Test 1

Draw a space diagram for three coplanar (in the same plane) forces all acting outwards from one point. The forces are 90 N due east, 120 N at 30° east of north and 60 N at 20° east of south.



(Ans.)

Coplanar Vectors

If the vectors are all in the same plane, they are called coplanar vectors.

Concurrent Vectors

Vectors whose lines of action meet at one point are called concurrent vectors. The forces shown in Figure 5 are coplanar, concurrent forces.

Resultant

The resultant of a system of vectors is the single vector that can replace the system of vectors and have the same effect. The resultant may be found by drawing a vector diagram and measuring the resultant (graphical method), or by calculation (analytical method).

Drawing a Vector Diagram

A vector diagram shows the vectors to scale with the angles accurately drawn, as in a space diagram. However, the positions of the vectors are shifted so that the head of one vector joins the tail of the next vector to form a continuous path.

To draw a vector diagram:

a) Select a suitable scale.

b) Take the vectors in order and draw them head to tail in a continuous path.

Example 1

Find the resultant of 3 concurrent coplanar forces:

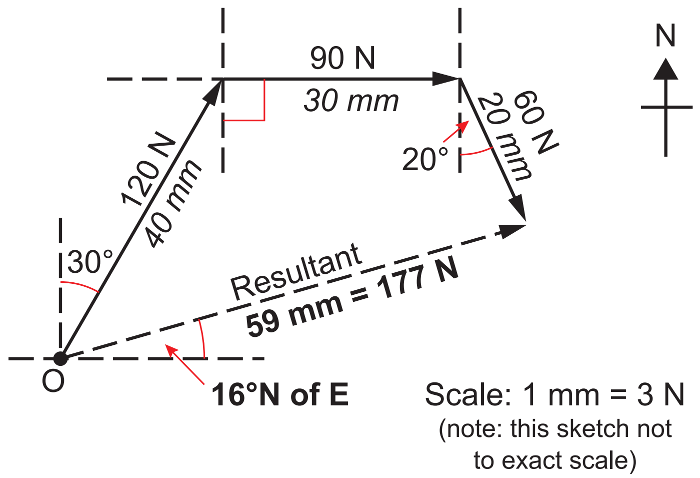
120 N acting 30° east of north

90 N acting east

60 N acting 20° east of south

The above forces are shown in the space diagram in the solution of Self-Test 1.

Figure 5 – Vector Diagram for Example 1



Solution 1

Refer to the space diagram in the solution of Self-Test 1 and Figure 5 for illustrations of the solution for Example 1.

1. A scale of 1 mm = 3 N is chosen for the vector diagram.
2. Show which direction is north in the diagram.
3. To draw the vector diagram, start at point O. A line is drawn angled 30° east of north and a distance of 40 mm is marked off to represent the 120 N force. An arrowhead is placed at the end of the vector to show the direction. The end of this arrow is the starting point for the next vector.
4. The next force is the 90 N force. A line is drawn, directed to the east, starting from the finishing point of the 120 N vector. A length of 30 mm is marked off to represent 90 N, and the arrowhead is drawn to show the direction of the force.
5. A line is then drawn starting from the finishing point of the 90 N vector and directed at an angle of 20° south of east. A length of 20 mm is marked off to represent 60 N. The arrowhead is drawn to show the direction of the force.

6. The resultant (shown dashed) is drawn from the starting point O, to the head of the last vector. Its measured length is 59 mm, which represents a magnitude of 177 N (59 × 3 N = 177 N). Its direction is 16° north of east from O, as measured with a protractor. The resultant's arrowhead must meet the last force's arrowhead; while the resultant's tail must be at the point of origin.

Note: Any of the forces could have been selected to start the vector diagram, and the forces could have been taken in any order. As long as the arrows follow head to tail, the result will be the same.

This method of finding the resultant by drawing a scale diagram is useful when checking the value of a resultant. However, it is not sufficiently accurate in most cases for problem solving.

To find the difference between vectors, add 180° to the angle being subtracted, and then add.

Therefore:

$$
\begin{aligned}
60^\circ - 45^\circ &= 60^\circ + (45^\circ + 180^\circ) \\
&= 225^\circ
\end{aligned}
$$

Self-Test 2

Add the following vectors and determine the resultants (length, angle from O).

a) 10 N, 120° + 5 N, 30°

b) 3 m/s, 210° + 10 m/s, 72°

c) 3 m/s, 45° + 5 m/s, 135°

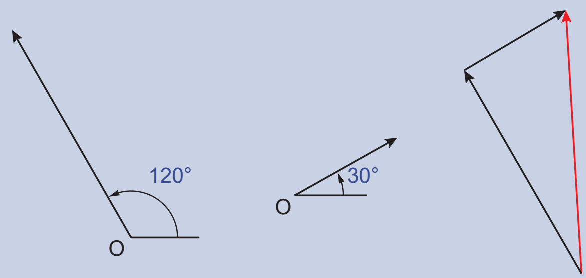
d) 3 m/s, 40° + 6 m/s, 125° + 3 m/s, 68°

e) 2.5 m/s, 25° + 3 m/s, 40° –5 m/s, 60°

Self-Test 2

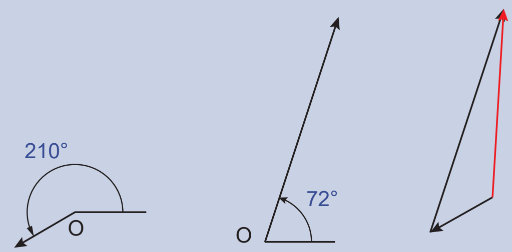
(Ans.)

A)



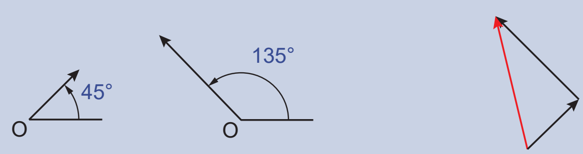
Resultant (in red)
11.18 N, 93.4°

B)



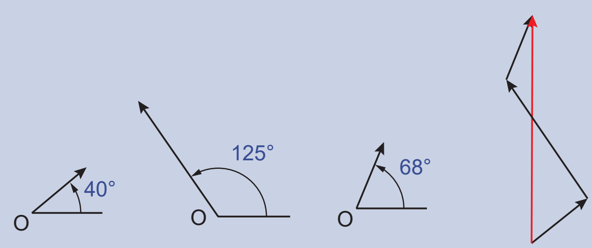
Resultant (in red)
8.03 m/s, 86.5°

C)



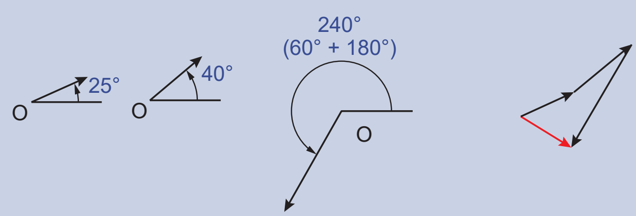
Resultant (in red)
5.83 N, 104.0°

D)



Resultant (in red)
9.62 m/s, 90.1°

E)



Resultant (in red)
2.46 m/s, -33.1°

Chapter Summary

This chapter showed that a scalar quantity only has a magnitude and that a vector quantity has both a magnitude and direction.

Notice that vectors, such as the velocity of an object, show both the speed (the scalar component) and the direction of the object (vector component). On the other hand, time only has a magnitude. Because time has no direction, it can only be a scalar quantity.

As shown in this chapter, vector quantities can come in many different forms, such as velocity, acceleration, and force. The one thing they share in common is that they all have a magnitude and a direction. Multiple vectors of the same kind, i.e. multiple forces, can be combined into one resulting vector. This can be determined by using a vector diagram.

CHAPTER 5

Linear Velocity and Acceleration

LEARNING OUTCOME

When you complete this chapter you should be able to:

Solve simple problems involving linear velocity, time, and distance.

LEARNING OBJECTIVES

Here is what you should be able to do when you complete each objective:

1. *Solve distance, displacement, speed, and velocity problems.*
2. *Draw graphs of velocity as a function of time.*
3. *Define acceleration, state its units, and solve simple acceleration problems.*
4. *Apply mathematical formulae relating acceleration, velocity, distance and time to solve problems.*

Chapter Introduction

There are two main areas of study involving motion. They are motion in a straight line (called linear motion) and motion in a circular path. This chapter will concentrate on linear motion only.

Acceleration is an important consideration when dealing with forces. It is the relationship between change in velocity and time and is a result of some force.

When dealing with velocity and acceleration, it is important to distinguish between scalar and vector quantities, and to understand Newton's laws of motion.

A scalar quantity is defined as a quantity that has magnitude only. A vector quantity has magnitude and direction.

Consider a 500 m walk to the local grocery store. If the same path was taken home, the total walking distance would be 1000 m (1 km); however, the total displacement after returning home would be zero This chapter will explain the reason why.

This chapter introduces practical examples and differences between scalar and vector quantities, such as distance vs displacement. It also focuses on velocity, average velocity, and acceleration.

A velocity-time graph will be introduced and its significance will be explained. From a single graph, one can gather a significant amount of information and an understanding of how an object is moving. As will be shown, the area under the curve is of particular interest.

Objective 1

Solve distance, displacement, speed, and velocity problems.

Distance and Displacement

Distance is a quantity that has magnitude only; therefore, it is a scalar quantity. For example, 100 km only refers to the length of the path over which a body travels; the direction is irrelevant.

Displacement refers to the change in position of a body, relative to a reference point (usually the initial staring point). Displacement has magnitude and direction; therefore, it is a vector quantity.

Example 1

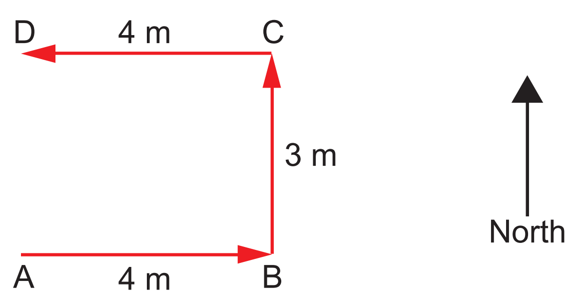
A body moves along a path from point A to point D as shown in Figure 1.

a) What distance does the body travel?

b) What is the displacement of the body when it moves to B from A?

c) What is the displacement of the body when it reaches D, from its original location A?

Figure 1 – Displaced Body



Solution 1

a) Total distance travelled from A to D:

4 m + 3 m + 4 m = **11 m (Ans.)**

b) Displacement of B from A:

4 m to the right of A, or due east (Ans.)

c) Displacement of D from A:

3 m above A, or due north (Ans.)

Note: The distance travelled does not always equal the displacement.

Speed and Velocity

Speed is the rate at which a body moves, which can also be stated as the distance moved in a given time. It is a scalar quantity since it is not concerned with the direction. The usual units of speed are metres per second (m/s) and kilometres per hour (km/h).

It is unlikely that speed will remain constant during a specific journey. The journey may be interrupted by stops, and the speed may increase or decrease along the way. Thus, it is usual to calculate the average speed in most cases.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Total time taken}}$$

Example 2

A vehicle travels a total distance of 300 km. The journey involves a total driving time of 6 hours, plus various stops taking 2 hours. What is the average journey speed?

Solution 2

$$\text{Journey time} = 6\text{ hours} + 2\text{ hours}$$

$$= 8\text{ hours}$$

$$\text{Average journey time} = \frac{300\text{ km}}{8\text{ hrs}}$$

$$= \mathbf{37.5\ km/h\ (Ans.)}$$

No direction, starting point or destination was given in this problem. It would not be possible to represent this journey graphically (by vectors). The term average speed is used to indicate that a scalar quantity is being dealt with.

Velocity refers to the speed of an object in a given direction, and is therefore a vector quantity, having magnitude (speed) and direction.

$$\text{Velocity} = \text{Speed} + \text{Direction}$$

An advantage of vector quantities is that they can be drawn to scale (in a diagram), and then added or subtracted graphically. Vector quantities contain more information than scalar quantities.

When considering velocities, it is normal to account for variations that may occur by considering average velocity.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

This formula can be rearranged as:

$$\text{Displacement} = \text{Average velocity} \times \text{Time}$$

The units of velocity are the same as the units of speed (m/s, km/h), with a direction added.

Note: When there is no change in the direction of a body in motion, then displacement and distance will be the same. Average speed and average velocity will also be the same.

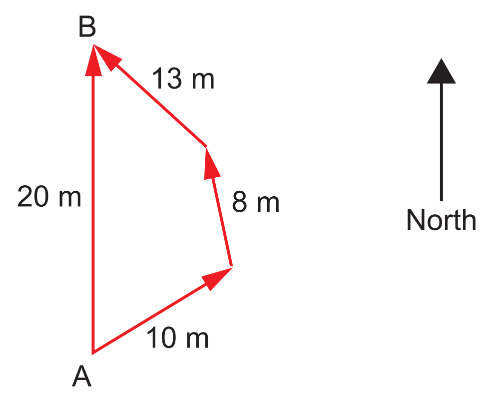
Example 3

A body moves along the path as shown in Figure 2, from Point A to Point B in a time of 10 s. Determine:

a) the average speed

b) the average velocity

Figure 2 – Path of Body



Solution 3

$$\text{Distance travelled} = 10\text{ m} + 8\text{ m} + 13\text{ m}$$

$$= 31\text{ m}$$

$$\text{Displacement of B to A} = 20\text{ m due north}$$

a) $$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time}}$$

$$= \frac{31\text{ m}}{10\text{ s}}$$

$$= \mathbf{3.1\ m/s\ (Ans.)}$$

b) $$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{20\text{ m}}{10\text{ s}}$$

$$= \mathbf{2\ m/s\ north\ (Ans.)}$$

Example 4

A vehicle travels 60 km due east in a time of 45 minutes. What is its average velocity in km/h?

Solution 4

$$\text{Displacement} = 60 \text{ km east}$$

$$\text{Time} = 45 \text{ minutes} = \frac{45 \text{ min}}{60 \text{ min/h}} = 0.75 \text{ h}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{60 \text{ km}}{0.75 \text{ h}}$$

$$= \mathbf{80\ km/h\ east\ (Ans.)}$$

Self-Test 1

A coal train travels 127 km from city A to city B which is 100 km due north of city A. If the average velocity of the train is 90 km/h north, how long will it take to make the journey?

1.11 hr (Ans.)

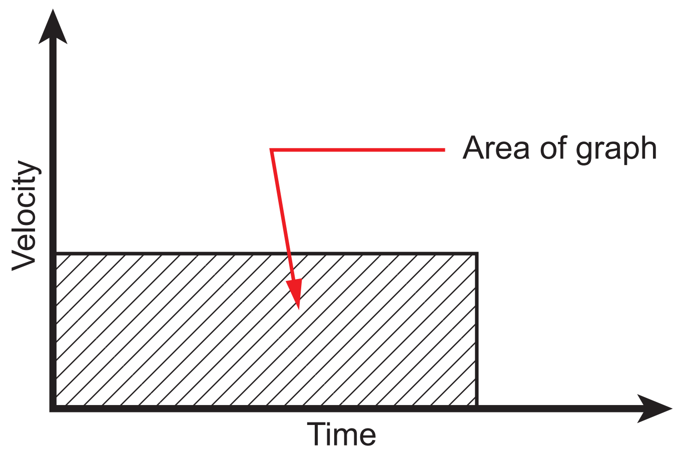
Objective 2

Draw graphs of velocity as a function of time.

Graphical Representation

A velocity versus time graph can be plotted to represent the movement of a body as follows:

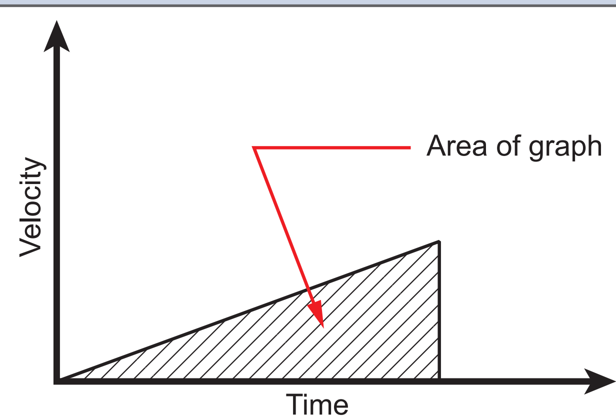
Figure 3 – Constant Velocity



$$\text{Area under graph} = \text{Velocity} \times \text{time}$$
$$= \text{Displacement}$$

The area under the line of a velocity-time graph represents displacement.

Figure 4 – Uniformly Increasing Velocity from Zero



$$\text{Area under graph} = \frac{1}{2}\text{base} \times \text{height}$$
$$= \text{Average velocity} \times \text{time}$$
$$= \text{Displacement}$$

Again, the area under the velocity-time graph is equal to the displacement.

Note: There are differences in technical definitions and distinction between speed and velocity. However, it is often common practice to use the term "velocity" interchangeably with the term "speed".

Where motion occurs in a straight line, and where consideration of direction is not important (does not affect the results of a calculation), the term velocity may be used in place of speed. However, if direction is important, then the velocity will include a direction (e.g. 50 m/sec, 45° north of east). That direction must be taken into account.

Also, if direction is a straight line, then the velocity-time graph could be called a speed-time graph. The area under the graph would then represent distance travelled. Remember that for linear travel (straight line) distance equals displacement.

Example 5

A train starts from rest and reaches a velocity of 120 km/h in 10 minutes. If the velocity increased uniformly, how far will the train travel in the first 10 minutes? Check calculations using a velocity-time graph.

Solution 5

Since the acceleration was uniform:

$$\text{Average velocity} = \frac{\text{Initial velocity} + \text{final velocity}}{2}$$

$$= \frac{0 + 120 \text{ km/h}}{2}$$

$$= 60 \text{ km/h}$$

$$\text{Time taken} = 10 \text{ min}$$

$$= \frac{10 \text{ min}}{60 \text{ min/h}}$$

$$= \frac{10}{60} \text{ h}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$\text{Displacement} = \text{Average velocity} \times \text{Time}$$

$$= 60 \text{ km/h} \times \frac{10}{60} \text{ h}$$

$$= \frac{600}{60} \text{ km}$$

$$= \mathbf{10 \text{ km (Ans.)}}$$

Drawing a velocity-time graph (Figure 5)

$$\text{Area of graph} = \text{Displacement}$$

$$\text{Area of graph} = \frac{1}{2}\text{ base} \times \text{height}$$

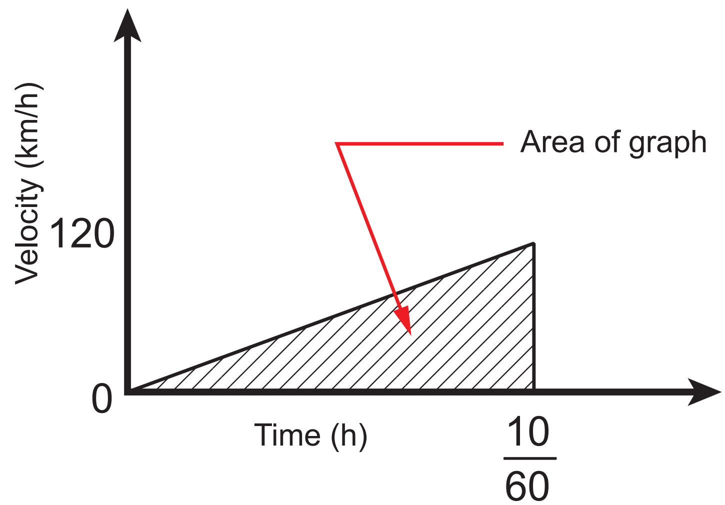
$$= \frac{1}{2} \times \frac{10}{60}\text{ h} \times 120\text{ km/h}$$

$$= \frac{1200}{120}\text{ km}$$

$$= \mathbf{10\ km\ (Ans.)}$$

Note: When drawing the graph, velocity and time plotted on the time scale must be the same: km/h and hours, m/s and seconds.

Figure 5 – Velocity-Time Graph



Mathematical Formula for Linear Velocity

A formula can be derived for bodies moving with uniform linear velocity as follows:

Let Initial velocity = u (m/s)

Final velocity = v (m/s)

Time = t (s)

Displacement = s (m)

From previous work:

$$\text{Displacement} = \text{Average velocity} \times \text{Time}$$

$$\text{Average velocity} = \frac{u + v}{2}$$

Therefore

$$s = \frac{u + v}{2} \times t$$

Example 6

A vehicle starts from rest and reaches a velocity of 60 km/h in 20 seconds. If it increases velocity uniformly, how far will it travel in the first 20 seconds? Give your answer in metres.

Solution 6

Initial velocity, u = 0

Final velocity, v = 60 km/h

Time, t = 20 s

$$60 \text{ km/h} = \frac{60 \text{ km/h} \times 1000 \text{ m/km}}{60 \text{ s/min} \times 60 \text{ min/h}}$$

$$= \frac{60\ 000}{3600} \text{ ms}$$

$$= 16.67 \text{ ms}$$

$$s = \frac{u + v}{2} \times t$$

$$= \frac{0 \text{ m/s} + 16.67 \text{ m/s}}{2} \times 20 \text{ s}$$

$$= \frac{16.67 \text{ m/s}}{2} \times 20 \text{ s}$$

$$= 8.335 \text{ m/s} \times 20 \text{ s}$$

Distance travelled = **166.7 m (Ans.)**

Self-Test 2

A vehicle traveling at 100 km/h decelerates uniformly to 60 km/h in 10 seconds. How far will the vehicle travel in this time? Answer in metres.

222.2 m (Ans.)

Objective 3

Define acceleration, state its units, and solve simple acceleration problems.

Acceleration

Acceleration may be defined as the rate of change of velocity or the change in velocity per unit time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken to change}}$$

$$= \frac{\text{m/s}}{\text{s}}$$

$$= \text{m/s}^2 \text{ (meters per second/second)}$$

A body is said to be accelerating if its velocity is changing with time. The change can be in speed (magnitude), direction or both.

An increase in speed is usually considered positive acceleration, while a decrease in speed is negative acceleration or deceleration.

Example 7

A car traveling at 20 km/h increases its velocity uniformly to 60 km/h in 10 seconds. What is the acceleration?

Solution 7

$$\text{Change in velocity} = 60 \text{ km/h} - 20 \text{ km/h}$$

$$= 40 \text{ km/h}$$

$$40 \text{ km/h} = \frac{40 \text{ km/h} \times 1000 \text{ m/km}}{60 \text{ s/min} \times 60 \text{ min/h}}$$

$$= \frac{40\ 000}{3600} \text{ m/s}$$

$$= 11.1 \text{ m/s}$$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$$

$$= \frac{11.1 \text{ m/s}}{10}$$

$$= \mathbf{1.11\ m/s^2\ (Ans.)}$$

Mathematical Formula for Linear Acceleration

Initial velocity $= u$ (m/s)

Final velocity $= v$ (m/s)

Time $= t$ (s)

Acceleration $= a$ (m/s^2)

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a = \frac{v - u}{t}$$

Using this formula to solve the problem in Example 7:

$$\begin{aligned}\text{Change in velocity} &= v - u \\ &= 60 \text{ km/h} - 20 \text{ km/h} \\ &= 40 \text{ km/h} \\ &= 11.1 \text{ m/s}\end{aligned}$$

$$\begin{aligned}a &= \frac{v - u}{t} \\ &= \frac{11.1 \text{ m/s}}{10 \text{ s}} \\ &= \mathbf{1.11\ m/s^2\ (Ans.)}\end{aligned}$$

A more familiar form for the equation $a = \frac{v - u}{t}$ is found by transposing:

$$a \times t = v - u$$

$$at + u = v$$

and $$v = u + at$$

Self-Test 3

A car traveling at 60 km/h is brought to rest over a distance of 100 m. What is the acceleration? Give the answer in m/s^2.

$-1.39\ m/s^2$ (Ans.)

Objective 4

Apply mathematical formulae relating acceleration, velocity, distance and time to solve problems.

Other Mathematical Formulae

Various formulae for velocity and uniform acceleration calculations can be derived from the two basic formulae.

$$s = \frac{u + v}{2} \times t$$

$$v = u + at$$

These formulae can be used to simplify many calculations.

Derived Formulae

$$s = \frac{u + v}{2} \times t$$

$$t = \frac{2s}{u + v}$$

Substituting this value for t in the formula v = u + at:

$$v = u + a \times \frac{2\,s}{u + v}$$

$$= u + \frac{2\,as}{u + v}$$

$$v - u = \frac{2\,as}{u + v}$$

$$(v - u)(v + u) = 2\,as$$

$$v^2 - u^2 = 2\,as$$

$$v^2 = u^2 + 2\,as$$

Formula for Distance

$$v = u + at \qquad \text{Equation (1)}$$

$$s = \frac{u + v}{2} \times t \qquad \text{Equation (2)}$$

Substituting u + at for v in Equation (2):

$$s = \frac{u + v}{2} \times t$$

$$= \frac{u + u + at}{2} \times t$$

$$s = \frac{2\,u + at}{2} \times t$$

$$= (u + \frac{1}{2}at)\,t$$

$$= ut + \frac{1}{2}at^2$$

Summary of Formulae for Uniform Linear Motion

$$s = \frac{u + v}{2} \times t$$

$$v = u + at$$

$$v^2 = u^2 + 2\,as$$

$$s = ut + \frac{1}{2}at^2$$

Example 8

An object is dropped from a height of 30 m. What will its velocity be on impact?

Solution 8

$v^2 = u^2 + 2\,as$

a = acceleration due to gravity

$= 9.81\ m/s^2$

u = initial velocity

$= 0\ m/s$

s = distance travelled

$= 30\ m$

$v^2 = u^2 + 2\,as$

$v^2 = 0 + 2 \times 9.81\ m/s^2 \times 30\ m$

$v^2 = 588.6\ m^2/s^2$

$v = \sqrt{588.6\ m^2s^2}$

Velocity on impact = **24.26 m/s (Ans.)**

Self-Test 4

A car has an initial speed of 20 km/h. In 1 minute and 30 seconds, it accelerates to 100 km/h. What is the distance travelled in metres during the acceleration period?

1500 m (Ans.)

Chapter Summary

Often, terms such as speed and velocity will be interchanged, as will distance and displacement. It is important to understand that these terms are not the same, and that scalar and vector quantities need to be treated differently. This chapter discussed the concepts of displacement, velocity, and acceleration; and how these three are all related.

As was shown, the area under a velocity-time graph is the total displacement. If the velocity is constant (i.e. the acceleration is 0), then the graph will be a simple rectangle, and the area will be calculated as $s = v \times t$. However, if the velocity is increasing at a constant rate (i.e. the acceleration is constant), the area will be represented by a triangle. The area then will be

$s = \frac{1}{2}$ base × height, or $s = \frac{u + v}{2} \times t$.

Understanding how these two equations are derived, why it is that they represent displacement, and how they are different is very important. Take the time to go back, if needed, to understand the graphical representation of these two equations.

It is important to understand that the problems in this chapter deal with constant acceleration only. The equations in **Objective 4** are only valid for problems with constant acceleration. Also, notice that all equations in **Objective 4** can be derived with two simple equations, using average velocity and the definition of acceleration.

CHAPTER 6

Force, Work, Pressure, Power, and Energy

Learning Outcome

When you complete this chapter you should be able to:

Perform calculations involving force, work, pressure, power, and energy.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Perform calculations involving force and work.*
2. *Perform calculations involving gauge, atmospheric, and absolute pressure.*
3. *Perform calculations involving power and different forms of mechanical energy.*

Chapter Introduction

Energy is neither created nor destroyed. One form of energy always converts to another. If energy is never destroyed, then why are all machines less than 100% efficient? That is because what appears as "wasted" energy inevitably turns into heat, which is another form of energy.

All processes that create friction take mechanical energy and transform it into heat. For example, when applying the brakes on a moving car, the kinetic energy of the car eventually converts into heat, through the process of applying friction.

This chapter deals with energy in its various forms, and how to convert energy into useful work. In order to do work, a body or a machine must be able to apply a force over a distance. The faster an object can be moved by applying a force, the more powerful it is.

A section of this chapter will cover the idea of pressure and the different types of pressure (gauge and absolute). In everyday work, it will likely be required to read and record gauge pressure values. However, if calculations are required, most steam tables will give values in absolute pressure. That is why it is important to understand the difference between the two. This chapter also addresses how work relates to power.

As an interesting example, consider a computer. The electronic components inside require electrical energy to operate, yet they do not produce any physical work. This means that the electrical energy converts into another form of energy. As is usually the case, energy that is not used to do work, turns up as waste heat. Notice when a computer runs for extended periods of time, the computer enclosure heats up. This concept will be covered when studying Thermodynamics.

Objective 1

Perform calculations involving force and work.

Force

A force is a push or pull exerted on an object. When a force is applied to an object, it may change the object's state of motion or rest, its location, its shape or its size. For example, sufficient force on a door will cause it to open; greater force causes it to open faster. If the door does not have a spring type closer or a doorstop, another force will have to be applied to stop the door from banging into a fixed object such as a wall. A bowling ball can apply a force to stationary bowling pins which sends them flying. The force of brakes can reduce the speed of an automobile. The force of a finger pressed against an inflated balloon can cause the balloon to become deformed.

In the SI system, the unit of force is called the newton (N).

$$1 \text{ N} = 1 \text{ kgm/s}^2$$

Variable Force

In practice, the magnitude of a force is not always constant, but may vary during its application. The force required to keep a vehicle moving will be constant when all conditions are constant. However, consider a typical highway with level sections and hilly sections. The added force required to move a vehicle up a hill is evident by the need to press on the accelerator. Downhill travel requires less force and in some cases, braking. Pedaling a bicycle into a strong head wind requires more force than pedaling with the wind behind.

Work

If force is applied to a body and causes it to move through a distance, then the work is done by the force. Work done by a force is the product of the force applied and the distance through which the force moves.

$$\text{Work done} = \text{Force} \times \text{Distance}$$

The work done when a force of one newton moves through a distance of 1 metre is one newton metre (Nm). This unit of work is called the joule (J).

$$1 \text{ joule} = 1 \text{ Newton metre (Nm)}$$

If a graph of force against distance moved is plotted, then the work done is represented by the area of the graph (Figure 1). Graphical representation very often simplifies work calculations.

Example 1

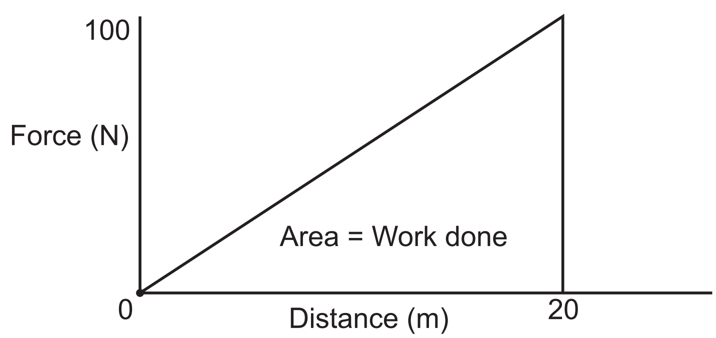
A force is uniformly increased from 0 N to 100 N to move a body a distance of 20 m. Calculate the work done and check the result graphically.

Solution 1 - Numerical

$$\text{Average force} = \frac{0\text{ N} + 100\text{ N}}{2}$$
$$= \frac{100\text{ N}}{2}$$
$$= 50\text{ N}$$
$$\text{Work done} = \text{Force} \times \text{Distance}$$
$$= 50\text{ N} \times 20\text{ m}$$
$$= 1000\text{ Nm}$$
$$= 1000\text{ J}$$
$$= \mathbf{1\ kJ\ (Ans.)}$$

Solution 1 - Graphical

Figure 1 – Force Distance Graph for Example 1



$$\text{Area of graph (triangle)} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 20\text{ m} \times 100\text{ N}$$
$$= \frac{2000}{2}\text{ Nm}$$
$$= 1000\text{ J}$$
$$= \mathbf{1\ kJ\ (Ans.)}$$

Example 2

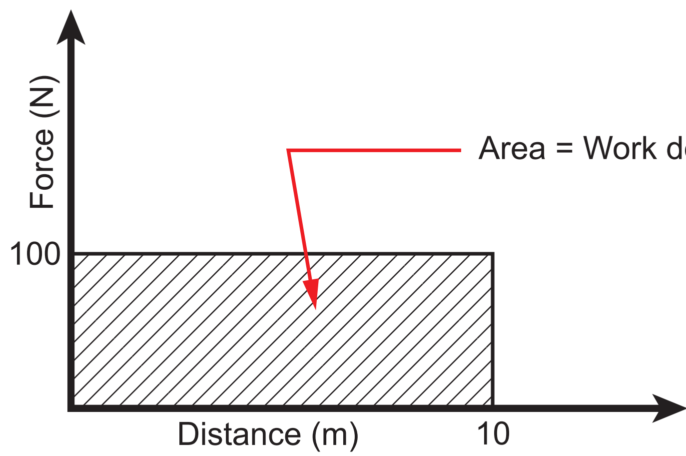
A constant force of 100 N acting on a body moves it a distance of 10 m. What is the work done by the force on the body?

Solution 2 - Numerical

$$
\begin{aligned}
\text{Work done} &= \text{Force} \times \text{Distance} \\
&= 100\ \text{N} \times 10\ \text{m} \\
&= 1000\ \text{Nm} \\
&= \mathbf{1\ kJ\ (Ans.)}
\end{aligned}
$$

Solution 2 - Graphical

Figure 2 – Force Distance Graph for Example 2



Example 3

A mass of 50 kg is moved a vertical distance of 20 m. What work will be required?

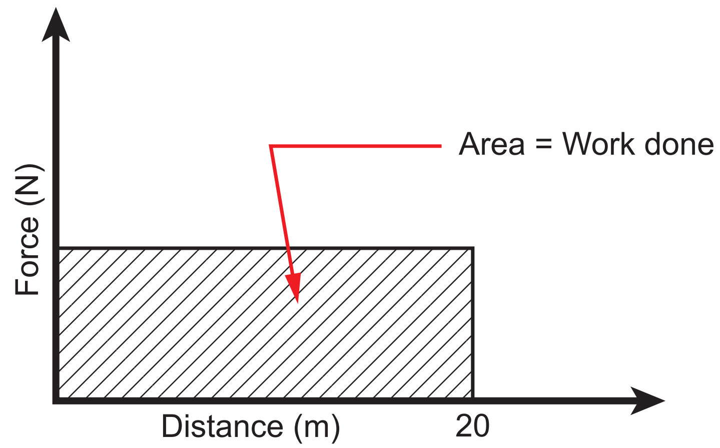
Solution 3 - Numerical

The force applied is the force necessary to overcome the force of gravity (50 kg × 9.81 m/s^2).

$$
\begin{aligned}
\text{Work done} &= \text{Force (N)} \times \text{Distance (m)} \\
&= (50\ \text{kg} \times 9.81\ \text{m/s}^2) \times 20\ \text{m} \\
&= (490.5\ \text{N}) \times 20\ \text{m} \\
&= 9810\ \text{Nm} \\
&= \mathbf{9.81\ kJ\ (Ans.)}
\end{aligned}
$$

Solution 3 - Graphical

Figure 3 – Force Distance Graph for Example 3



Self-Test 1

In Figure 4, a pump is required to raise 500 litres of fresh water a vertical distance of 100 m.

a) How much work will be done by the pump in raising the water?

b) If the pump has an efficiency of 50%, how much work must be done on the pump to raise the water?

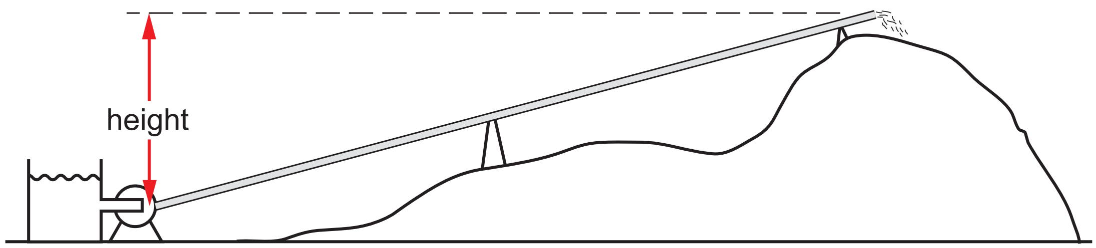
490.5 kJ (Ans. a)
981 kJ (Ans. b)

Note: Twice as much work is put into the pump as is done by the pump.

It should be noted here that the work done in pumping water uphill (Figure 4) is not a function of how long the pipe is, but only of the height of the point of pipe discharge measured vertically above the pump outlet.

Excessively long pipes or high velocity may reduce the efficiency of the system because of friction in the pipe.

Figure 4 – Work Done by Pump



The actual work done by raising an oil drum from the floor to a loading dock is a function of the drum's mass and the vertical height of the dock above the floor. A ramp from the floor could be four times as long as the vertical height of the dock, but the actual work performed in rolling a drum up the ramp is not four times as much as the vertical lift. The inclined plane of the ramp helps to raise the drum more slowly and with less force, but the work accomplished is still the same (neglecting small frictional losses).

A heavily loaded truck may travel at 100 km/h along a flat countryside. When the truck comes to a steep hill, the driver uses lower gears to move the heavy load up the hill, but at a much slower rate. Road, mechanical, and wind resistance all play a part in how efficiently the truck changes elevation. The end result is still that the mass is raised the vertical height of the hill, regardless of how long the road is.

Objective 2

Perform calculations involving gauge, atmospheric, and absolute pressure.

Pressure

The pascal (Pa) is the SI unit for pressure measurement. Pressure is a measure of force per unit area. Since force is measured in newtons and area is measured in m^2, one newton per square metre equals one pascal.

$$1 \text{ N/m}^2 = 1 \text{ Pa}$$

Or $$1 \text{ kN/m}^2 = 1 \text{ kPa}$$

A S

Example 4

If 350 N acts on an area of 0.25 m^2, what is the resulting pressure?

Solution 4

A S

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{350 \text{ N}}{0.25 \text{ m}^2}$$

$$= 1400 \text{ Pa}$$

$$= \mathbf{1.4\ kPa\ (Ans.)}$$

Self-Test 2

What force is exerted on a rectangular tank wall 1.3 m long and 0.75 m high when a pressure of 150 kPa acts upon it?

146.25 kN (Ans.)

Absolute and Gauge Pressure

The force due to the weight of the atmosphere above us produces **atmospheric pressure**. Assuming that the atmosphere forms an almost perfect sphere around the earth, the higher the elevation, the less atmosphere there is above us. Thus, atmospheric pressure decreases with elevation. A number of other factors relating to air conditions will also affect this pressure, such as:

- Moisture content (humidity)
- Density
- Temperature

However, on average, the pressure due to the atmosphere at sea level has been measured as 101.3 kPa. This pressure is measured relative to a theoretical "absolute zero" of pressure and to be precise, atmospheric pressure is 101.3 kPa **absolute**. In practice, the term "absolute" is usually omitted.

Since atmospheric pressure is always present on earth, its presence is discounted. Usually, pressure gauges begin at a value of 0 kPa. Pressures measured with these gauges are actually **gauge** pressures. If the absolute pressure of a substance is required, the following equation can is used.

Absolute pressure = Gauge pressure + Atmospheric pressure

In thermodynamics, it is important to know the absolute pressure of a system as all pressure tables are quoted in these units.

Example 5

If the pressure gauge on a steam drum reads 1200 kPa, what is the absolute pressure inside the drum?

Solution 5

Absolute pressure = Gauge pressure + Atmospheric pressure

= 1200 kPa + 101.3 kPa

= **1301.3 kPa (Ans.)**

Objective 3

Perform calculations involving power and different forms of mechanical energy.

Power

Power is the rate of doing work or the quantity of work done per unit of time.

$$\text{Power} = \frac{\text{Work done}}{\text{Time}}$$
$$= \frac{\text{Nm}}{\text{s}}$$
$$= \frac{\text{Joule}}{\text{s}}$$

The unit of power J/s is called the watt (W).

$$1\ \text{J/s} = 1\ \text{watt}$$

Since the watt is a small unit of power, the usual unit of power for most applications is the kilowatt.

$$1\ \text{kilowatt (kW)} = 1000\ \text{watt}$$

A megawatt is used for very large power outputs.

$$1\ \text{megawatt (MW)} = 1\,000\,000\ \text{watts}$$
$$= 1000\ \text{kW}$$

Example 6

A mass of 50 kg is moved a vertical distance of 20 m in 10 s. What is the power developed?

Solution 6

$$\text{Work done} = \text{Force (N)} \times \text{Distance (m)}$$
$$= (50\ \text{kg} \times 9.81\ \text{m/s}^2) \times 20\ \text{m}$$
$$= (490.5\ \text{N}) \times 20\ \text{m}$$
$$= 9810\ \text{Nm}$$
$$= 9810\ \text{J}$$

$$\text{Power developed} = \frac{\text{Work done}}{\text{Time}}$$
$$= 9810\ \text{J}/10\ \text{s}$$
$$= 981\ \text{J/s}$$
$$= \mathbf{981\ W\ (Ans.)}$$

Self-Test 3

A pump raises 200 tonnes of fresh water a height of 10 m in 1 hour. What is the power output of the pump?

5.45 kW (Ans.)

Example 7

If the efficiency of the pump in Self-Test 3 is 80%, what is the power required to drive the pump?

Solution 7

$$\%\ \text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} \times 100\%$$

$$\text{Power input} = \frac{\text{Power output}}{\%\ \text{Efficiency}} \times 100\%$$

$$= \frac{5.45\ \text{kW}}{80\%} \times 100\%$$

$$= \mathbf{6.81\ kW\ (Ans.)}$$

In the Imperial system, the unit of power is called the horsepower. This unit is still often used to rate electric motors.

$$1\ \text{horsepower} = 746\ \text{W}$$

Since $\text{Power} = \frac{\text{Work}}{\text{Time}}$, then

$$\text{Work} = \text{Power} \times \text{Time}$$

Work is sometimes expressed in terms of Power × Time.

For example:

1 Kilowatt hour = Work done when 1 kilowatt of power is exerted for 1 hour

$$
\begin{aligned}
1\ \text{kWh} &= 1\ \text{kW} \times 1\ \text{h} \\
&= 1000\ \text{W} \times 3600\ \text{s} \\
&= 1000\ \text{J/s} \times 3600\ \text{s} \\
&= 3\ 600\ 000\ \text{j} \\
&= 3.6\ \text{MJ}
\end{aligned}
$$

Energy

A body possesses energy when it is capable of doing work; thus, energy is the capacity to do work. The basic SI unit of energy is the joule, the unit of work when dealing with such forms of energy as:

- Thermal
- Mechanical

Thermal Energy

Thermal energy is expressed in J, kJ, MJ, or GJ in the SI system.

Mechanical Energy

There are two forms of mechanical energy.

- Potential
- Kinetic

Potential Energy

Potential energy (PE) is the ability of a body to do work due to its position. For example, water stored in a dam contains potential energy due to its position and can be made to do such work as turning a hydroelectric alternator. If the water is released, it will flow to a lower point due to the force of gravity.

$$
\begin{aligned}
\text{PE} &= \text{Gravitational force (newtons)} \times \text{Vertical height (metres)} \\
&= \text{Mass} \times g \times h, \qquad \text{where } g \text{ is the acceleration due to gravity}
\end{aligned}
$$

A S

Example 8

An object with a mass of 100 kg is at a height of 10 m above the ground. What is its potential energy?

Solution 8

$$
\begin{aligned}
PE &= \text{Mass} \times \text{Acceleration due to gravity} \times \text{height} \\
&= 100 \text{ kg} \times 9.81 \text{ m/s}^2 \times 10 \text{ m} \\
&= 9810 \text{ Nm} \\
&= \mathbf{9810 \text{ J (Ans.)}}
\end{aligned}
$$

Kinetic Energy

Kinetic energy (KE) is the ability of a body to do work due to its motion.

$$\text{Mass of the body} = m \text{ (kg)}$$

$$\text{Velocity of the body} = v \text{ (m/s)}$$

Then

$$KE = \frac{1}{2} mv^2 \text{ (J)}$$

AS

Example 9

A body with a mass of 100 kg is moving at a velocity of 100 km/h. What is the kinetic energy?

Solution 9

$$
\begin{aligned}
m &= 100 \text{ kg} \\
v &= \frac{100 \text{ km} \times 1000 \text{ m/km}}{1 \text{ h} \times 60 \text{ min/h} \times 60 \text{ s/min}} \\
&= \frac{100 \times 1000 \text{ m}}{3600 \text{ s}} \\
&= 27.78 \text{ m/s}
\end{aligned}
$$

$$
\begin{aligned}
KE &= \frac{1}{2} mv^2 \\
&= \frac{1}{2} \times 100 \text{ kg} \times (27.78 \text{ m/s})^2 \\
&= \frac{1}{2} \times 100 \text{ kg} \times 771.73 \text{ m}^2/\text{s}^2 \\
&= 38\,586.5 \text{ J} \\
&= \mathbf{38.5865 \text{ kJ (Ans.)}}
\end{aligned}
$$

Transformation of Mechanical Energy

Potential energy can be transformed to kinetic energy. The law of conservation of energy states that energy cannot be created or destroyed but can change its form.

Loss in PE = Gain in KE (or vice versa)

This is a useful relationship for solving many problems.

Example 10

A resting block with a mass of 50 kg is allowed to fall, from rest, from a height of 10 m. What will be its velocity upon striking the ground?

Solution 10

$$\begin{aligned}\text{Potential energy} &= \text{Mass} \times \text{Acceleration due to gravity} \times \text{Height} \\ &= 50\ \text{kg} \times 9.81\ \text{m/s}^2 \times 10\ \text{m} \\ &= 4905\ \text{Nm} \\ &= \mathbf{4905\ J\ (Ans.)}\end{aligned}$$

After falling, its entire PE is transformed into KE, due to the block being at zero height above the ground.

$$\begin{aligned}\text{Loss in PE} &= \text{Gain in KE} \\ 4905\ \text{J} &= \frac{1}{2} \times mv^2 \\ &= \frac{1}{2} \times 50\ \text{kg} \times v^2 \\ \frac{4905\ \text{J} \times 2}{50\ \text{kg}} &= v^2 \\ \frac{9810\ \text{J}}{50\ \text{kg}} &= v^2 \\ v^2 &= 196.2\ \text{J/kg} \\ &= 196.2\ \text{Nm/kg} \\ &= 196.2\ \text{kgm}^2/\text{s}^2/\text{kg} \\ &= 196.2\ \text{m}^2/\text{s}^2 \\ v &= \sqrt{196.2\ \text{m}^2/\text{s}^2} \\ &= \mathbf{14.01\ m/s\ (Ans.)}\end{aligned}$$

At the moment of striking the ground, the PE will be zero and the KE at maximum. The velocity will be 14.01 m/s when the block strikes the ground.

At 5 m above the ground, half of its PE will have been converted into KE. Thus, at 5 m:

$$PE = KE$$

$$PE = \frac{1}{2} \times 4905 \text{ J}$$

$$PE = 2452.5 \text{ J}$$

Self-Test 4

A 60 kg mass falls freely to the ground from rest. It hits the ground at 10 m/s. From what height did it fall?

5.1 m (Ans.)

Chapter Summary

The idea of variable force was a new concept introduced in this chapter. Note that when plotting a curve of force vs. distance, the area under the curve is the total work done. If the force is constant, then the calculation is simple, $W = F \times d$. However, if the force is not constant, the area under the curve becomes more complicated to calculate. That is why graphical representation is often very important.

The amount of work required to raise an object is only dependent on the mass of the object and the total height it is raised. Even if using a ramp, over a long distance, the total work required to raise the object is still only dependent on the total height raised. This is assuming the ramp is friction free.

Pressure is defined as the perpendicular force applied across an area. Consider how snow shoes work. Due to the snow shoes, a large force (body weight) is distributed over a larger area than just the feet. As a result, the net pressure is significantly less. This permits a person to walk on snow.

When dealing with pressure, it is important to understand the differences between gauge and absolute pressure. Absolute pressure is defined as the sum of the gauge and atmospheric pressure.

Mechanical energy is the ability of an object to do work. It may be in the form of kinetic or potential energy. Kinetic energy can be transformed into potential energy and vice versa. Work can also be expressed as power if the time frame is known. Power is the rate at which work is done.

CHAPTER 7

Friction

Learning Outcome

When you complete this chapter you should be able to:

Solve problems involving friction.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Apply the laws governing the types of friction.*
2. *Apply the coefficient of friction to problems involving forces on a horizontal plane.*

Chapter Introduction

Consider rubbing one hand with the other. In order to move hands against each other, it is necessary to overcome a force that prevents this motion. The preventative force is called friction and friction produces heat.

Friction is often considered a nuisance: something that needs to be reduced. However, that is not always the case. A car tire is a good example of an object where friction is both desired and undesired. In order to increase grip, high friction is preferred. In order to reduce fuel consumption, lower friction is preferred.

This chapter deals with different types of friction forces and the laws that govern them. It will show how to calculate the friction force by understanding the importance of the coefficient of friction.

Objective 1

Apply the laws governing the types of friction.

Friction

The "force of friction" can be defined as a force that opposes motion of one surface over another. This opposition to motion is due to the irregularities of the two surfaces. The direction of the friction force is opposite the direction of any force trying to move, or moving, an object.

The control of friction is necessary in many situations. Friction may be reduced by lubrication. This separates the two surfaces and results in only the resistance of the lubricant itself causing friction. An example is a rotating shaft, supported by an oil-supplied bearing. The oil, which is under pressure, causes the shaft to ride on a cushion of oil instead of allowing metal to metal contact.

Friction may be necessary in some cases. An increase in friction may be achieved by selecting the appropriate materials and by applying forces in the proper direction. An example is two pulleys joined by a belt. The selection of the belt material and belt tension may increase the friction. Friction can be an asset or a liability, depending upon the application.

Types of Friction

Friction occurs in several different forms.

Standing or static friction is the resistance that opposes the initial movement of a body at rest. In other words, to start an object moving, a certain amount of frictional resistance must be overcome. This resistance is called static friction.

Sliding or kinetic friction is the resistance that opposes the continued movement of an object. To keep an object moving at a constant speed requires a constant force to overcome the kinetic friction.

Rolling friction is the resistance that opposes the motion of a wheel or roller as it rolls along a surface. An example of this is a tire rolling over concrete.

Fluid friction is the resistance to movement within the layers of a fluid.

Friction always exists to some extent. A percentage of the energy input to any machine is consumed in overcoming frictional forces. The two main types of friction examined here will be static friction and kinetic friction.

Laws Governing Friction

The laws governing friction are not precise; however, they have been experimentally determined to hold true for any situation encountered.

a) The force of friction is proportional to the force that presses the two surfaces together. For static or kinetic friction, this will depend upon the mass of an object on a horizontal surface; more precisely the total force normal (perpendicular) to the surface. If the downward force is doubled, the frictional force between the two surfaces is doubled.

b) Static friction (standing friction) is always greater than kinetic friction (moving friction). The amount of force required to start an object moving is greater than that required to keep the object moving at a constant speed.

c) The force of friction, whether static or kinetic, is not affected by the area of the two surfaces in contact. If the same downward force is distributed over twice as much area, there is no change in the frictional forces between the two surfaces.

d) Kinetic friction is not affected by the speed of the body. Within reasonable limits, the forces of kinetic friction between two surfaces will remain unchanged as the speed of the object is increased or decreased.

e) The force of friction (either static or kinetic) is affected by the relative roughness of the two surfaces in contact. The rougher the surfaces, the greater the forces required to overcome friction. If a greater force of friction is desired, simply increase the roughness on one of the surfaces.

f) Kinetic friction (sliding friction) is greater than rolling friction, which explains why roller type bearings are extensively used.

Objective 2

Apply the coefficient of friction to problems involving forces on a horizontal plane.

Coefficient of Friction

The coefficient of friction (μ) is the ratio of the frictional force opposing a body's motion on a surface to the normal (i.e. perpendicular) reaction force between the body and the surface. The coefficient of friction is a ratio and, therefore, has no units. Stated mathematically,

$$\mu = F_F/R_N$$

Where

- μ is the coefficient of friction.
- F_F is the friction force (parallel to the surface). This force is also equal in magnitude to the force required to start an object moving, or to keep it moving at a constant speed. When dealing with static friction between horizontal forces and surfaces, at the instant the object moves, the applied force (F_A) equals the friction force (F_F). For kinetic friction between horizontal forces and surfaces, the applied force (F_A) equals the friction force (F_F) when an object is moving at constant speed.
- R_N is the reaction force between the surfaces, normal (perpendicular) to the surface. R_N is opposite in direction and equal in magnitude to the downward force of the object due to gravity, when dealing with horizontal surfaces.

Note: F_F and R_N must be at right angles to each other.

Table 1 shows some examples of the coefficient of friction between two surfaces. These values have been determined experimentally and should be considered as typical ranges.

Table 1 – Example of Coefficient of Friction

Surfaces	Coefficient of Friction
metal on metal	0.15 to 0.65
greased metal on metal	0.02 to 0.06
metal on wood	0.20 to 0.60
wood on wood - dry	0.25 to 0.55
wood on wood - wet	0.10 to 0.45
rubber on concrete	0.60 to 0.95

Figure 1 shows the forces acting on a block under static friction conditions. If force F_A is less than force F_F, then the block will not move. When force F_A is equal to force F_F, then the block will just start to move. If force F_A is increased, then the block will accelerate.

Figure 1 – Static Friction

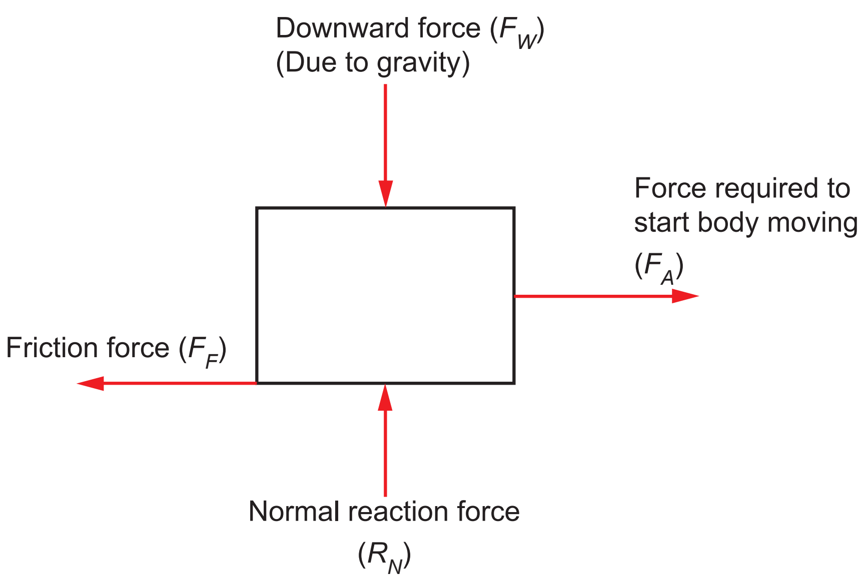
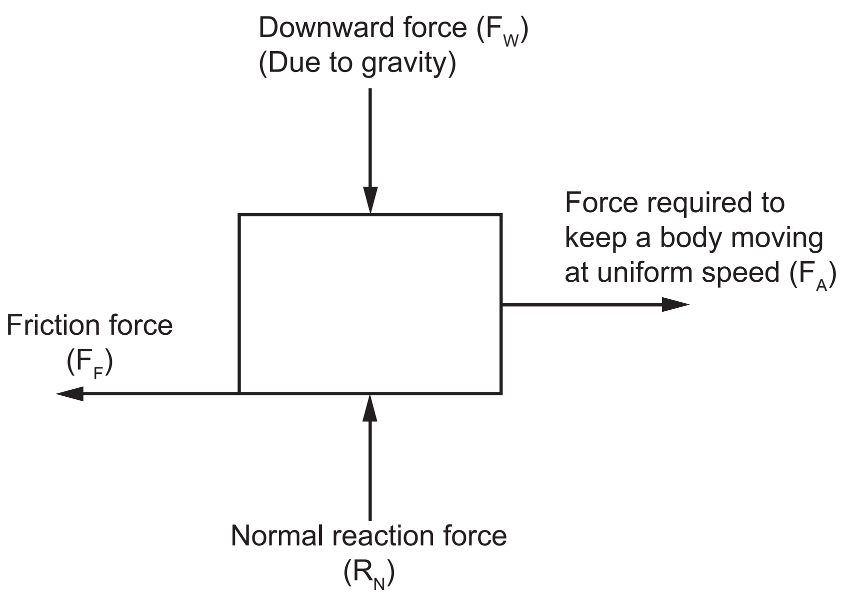


Figure 2 shows the forces acting on a block under kinetic friction conditions. If the force F_A is equal to the force F_F, then the block will move at constant velocity. If the force F_A is increased, then the block will accelerate.

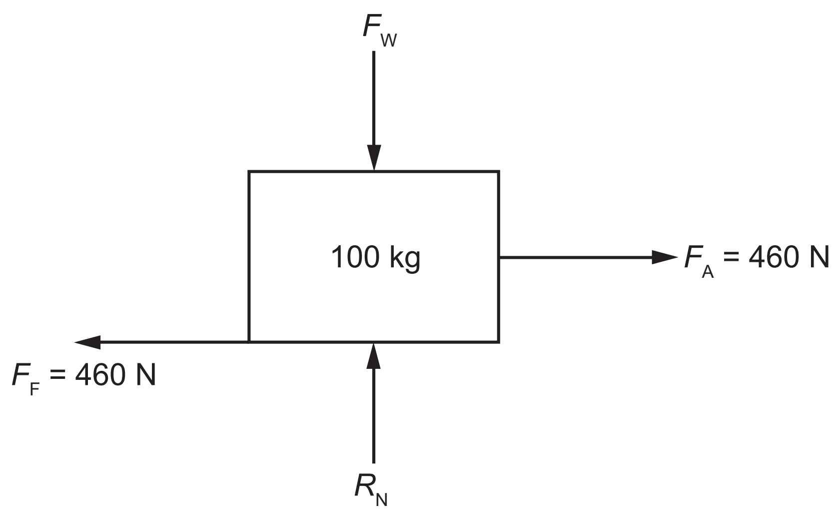
Figure 2 – Kinetic Friction



Example 1

Find the coefficient of friction between a 100 kg box (Figure 3) and a horizontal concrete floor. A horizontal force of 460 N is required to start the box moving.

Figure 3 – Coefficient of Friction Example



Solution 1

First, calculate R_N from the 100 kg mass. The mass (m), multiplied by the acceleration due to gravity (g), will be equal to the downward force (F_W). R_N is numerically equal to the downward force when the surface and applied force are both horizontal.

Therefore

$$\begin{aligned} F_W &= mg \\ &= 100\ \text{kg} \times 9.81\ \text{m/s}^2 \\ &= 981\ \text{N} \end{aligned}$$

Then

$$R_N = 981\ \text{N}$$

And

$$F_F = F_A = 460\ \text{N}$$

$$\text{Coefficient of static friction } (\mu_s) = \frac{F_F}{R_N}$$

$$\mu_s = \frac{460\ \text{N}}{981\ \text{N}}$$

$$= \mathbf{0.47\ (Ans.)}$$

Example 2

Find the coefficient of sliding (kinetic) friction if the box in Figure 3 requires a force of 430 N to keep it moving at constant speed.

Solution 2

$$R_N = 981 \text{ N}$$

$$F_F = F_A = 430 \text{ N}$$

$$\text{Coefficient of sliding friction } (\mu_k) = \frac{F_F}{R_N}$$

$$\mu_s = \frac{430 \text{ N}}{981 \text{ N}}$$

$$= \mathbf{0.44\ (Ans.)}$$

Note: The coefficient of static friction is greater than the coefficient of kinetic friction.

$$\mu_s > \mu_k$$

The subscripts "s" and "k" are not always included with "μ". The wording of a problem often indicates whether the frictional forces are static or kinetic.

Example 3

Find the horizontal force required to move a pump and the skid on which it rests. The total mass is 2000 kg. The coefficient of static friction between the horizontal floor and the skid is 0.26.

Solution 3

First, find the value of R_N

$$F_W = \text{Mass} \times \text{acceleration due to gravity}$$

$$= 2000 \text{ kg} \times 9.81 \text{ m/s}^2$$

$$= 19\,620 \text{ N}$$

Thus

$$R_N = 19\,620 \text{ N}$$

Now substitute in the equation

$$\mu_s = \frac{F_F}{R_N}$$

$$F_F = R_N \times \mu_s$$

$$= 19\,620 \times 0.26$$

$$= \mathbf{5101.2\ N\ (Ans.)}$$

Self-Test 1

Find the normal force and the mass of an object when a horizontal force of 2100 N just starts to move the object along a horizontal surface. The coefficient of static friction is 0.40.

5250 N, 535.2 kg (Ans.)

Chapter Summary

Friction force opposes motion. It always acts in the opposite direction of motion.

Four different types of friction forces were covered. These are standing, sliding, rolling, and fluid friction. The law of friction, formulated through experimentation, governs why certain types of friction forces are greater than others.

In order to calculate the friction force one must know the normal force, also known as the reaction force, which a surface applies to the object. The other requirement asks for the coefficient of friction to be determined by the two surfaces in contact. Notice that rubber in itself does not have a coefficient of friction; rather it would be a coefficient of friction between it and another material.

Chapter 8

Stress and Strain

Learning Outcome

When you complete this chapter you should be able to:

Explain physical properties of materials and how their behaviour is affected when external forces are applied.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Describe the mechanical properties of materials, including elasticity, stiffness, plasticity, ductility, toughness, brittleness, and hardness.*
2. *Calculate stress including tensile, compressive, and shear stresses within rigid bodies due to external loads.*
3. *Calculate the strain of members under load.*

Chapter Introduction

External forces applied to a body have a tendency to deform it. But the body develops an internal resistance against the deforming forces. The external forces acting on a rigid body are termed loads. The deformation is known as strain. The internal resistance to the external force is called stress.

The stress increases with additional external force up to a certain limit, beyond which the body will permanently deform or even break.. All materials subjected to stress will change their shape and become strained; though the amount of strain may be too small to measure.

Often times the terminology used to describe the mechanical properties of a material are used interchangeably and incorrectly. In this chapter, the different mechanical properties of a material will be summarized. Take note on what each property truly describes.

Consider a column used to support a building load. This column must be capable of holding this load without completely deforming. In order to understand how to design this column and what material to choose, one must have a good understanding of the stress-strain relationship. This chapter will discuss what stress is and how it induces strain. The differences between shear, tensile and compressive stress will be addressed.

As the chapter proceeds, note that it is not only the material property that is important, but also the geometry that matters. A stress at which a material fails is an inherent property. The stress resulting from a load to reach failure is also dependent on the cross sectional area of the material.

Objective 1

Describe the mechanical properties of materials, including elasticity, stiffness, plasticity, ductility, toughness, brittleness, and hardness.

Mechanical Properties of Materials

Mechanical properties are those characteristics of a material that can only be determined by the application of force. The ability of a material to withstand stress and strain depends on the characteristics of the material itself. To correctly specify a material for a certain application, a designer must know the following important mechanical properties of the material.

Elasticity

Elasticity is the ability of a material to return to its original shape after a deforming force is removed.

Stiffness

Stiffness is the ability of a material to resist a change in shape or size when a load is applied.

Plasticity

Plasticity is the ability of a material to retain its deformed shape when a deforming force is removed.

Ductility

Ductility is the ability of a material to be stretched and reduced in cross section without breaking. Ductile materials will undergo large deformations before breaking.

Toughness

Toughness is the ability of a material to absorb energy before breaking. Tough materials will absorb and dissipate energy by bending, twisting, and changing shape considerably before they break.

Brittleness

Brittle materials break without much deformation occurring before fracture.

Hardness

Hardness is the ability of a material to resist penetration.

Objective 2

Calculate stress including tensile, compressive, and shear stresses within rigid bodies due to external loads.

Stress

Stress is defined as the internal resistance of a material to an external force (load) that is being applied. Stress is expressed as the applied force (load) per unit area of a material.

$$\text{Stress} = \frac{\text{Load (N)}}{\text{Area (m}^2\text{)}}$$

In SI units, stress is indicated in pascals, Pa.

$$1\ \text{N/m}^2 = 1\ \text{pascal}$$

Pascals are very small units. Larger units such as kilopascals (kPa), megapascals (MPa), and gigapascals (GPa) are more commonly used.

Theoretically, stresses act in all directions. For simplicity, the stresses are resolved into two components. One has a direction perpendicular to the surface being considered. The other is parallel to this surface.

Perpendicular forces cause compressive or tensile stress, depending on whether they push or pull on the surface being considered. Forces that are parallel to a surface cause shear stress.

Mathematically, stress is expressed as:

$$\sigma = \frac{P}{A}$$

Or

$$\tau = \frac{P}{A}$$

Where

σ = Normal stress intensity – pascals

τ = Parallel (shear) stress intensity – pascals

P = Perpendicular or parallel load applied – newtons

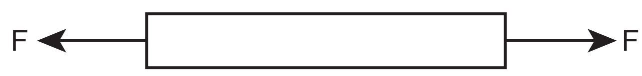
A = Cross-sectional area of the section being considered – square metres

The ultimate stress, (the stress at which the material breaks) is defined as the maximum load, at which breakage occurs, divided by the original cross-sectional area of the material.

Tensile Stress

Consider a straight bar of uniform cross section (Figure 1) subjected to a pair of forces acting in opposite directions and coinciding with the axis of the bar. If the forces are directed away from the bar, they tend to increase the length of the bar, which is said to be in tension. The internal stress developed (tensile stress) is normal to the cross-section and the bar is called a tie.

Figure 1 – Member Under Tension



Example 1

A round tie bar is subjected to an axial (along its length) load of 100 kN. If the diameter of the tie is 0.050 m, what is the stress on the tie?

Solution 1

a) Load = 100 kN or 100 000 N

Calculate the area of the tie:

$$A = \frac{\pi d^2}{4}$$

$$= \frac{3.1416 \times (0.05)^2 m^2}{4}$$

$$= \frac{3.1416 \times 0.0025\ m^2}{4}$$

$$= \frac{0.0079\ m^2}{4}$$

$$= 0.001\ 963\ m^2$$

b) Calculate the stress, σ:

$$\sigma = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{100\ 000\ N}{0.001\ 963\ m^2}$$

$$= 50\ 942\ 435\ Pa$$

$$= \mathbf{50\ 942.435\ kPa\ (Ans.)}$$

Compressive Stress

If the forces described in Figure 1 are directed towards the bar as in Figure 2, they will tend to compress the material. Then the bar is said to be in compression; the internal stress will be a compressive stress and the member is called a strut.

Figure 2 – A Member Under Compression

Example 2

A compressive load of 107 kN is resisted by a rectangular strut that is 50 mm × 75 mm. What is the stress in the strut?

Solution 2

$$\text{Area, A} = 0.05\text{ m} \times 0.075\text{ m}$$

$$= 0.003\ 75\text{ m}^2$$

$$\text{Stress, } \sigma = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{107\ 000\text{ N}}{0.003\ 75\text{ m}^2}$$

$$= 28\ 533\ 533\text{ Pa}$$

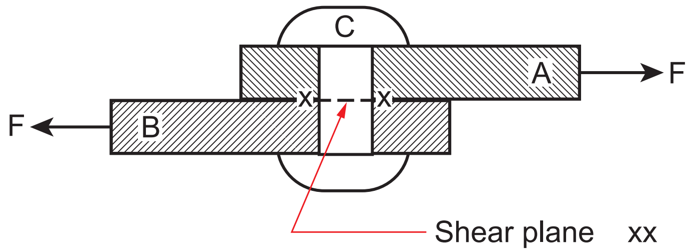
$$= \mathbf{28\ 533\ kPa\ (Ans.)}$$

Shear Stress

If the external forces act parallel to one another, but are not in the same plane, a shear stress will result. Consider two plates (Figure 3), A and B, joined together by a rivet, C. If a tensile load, F, is applied to the plates, the force or load is exactly parallel to the cross-section of the rivet (x-x). If overloaded, the rivet would shear along its cross section (x-x). If d is the diameter of the rivet, the cross-sectional area of the rivet resisting the shear force P is:

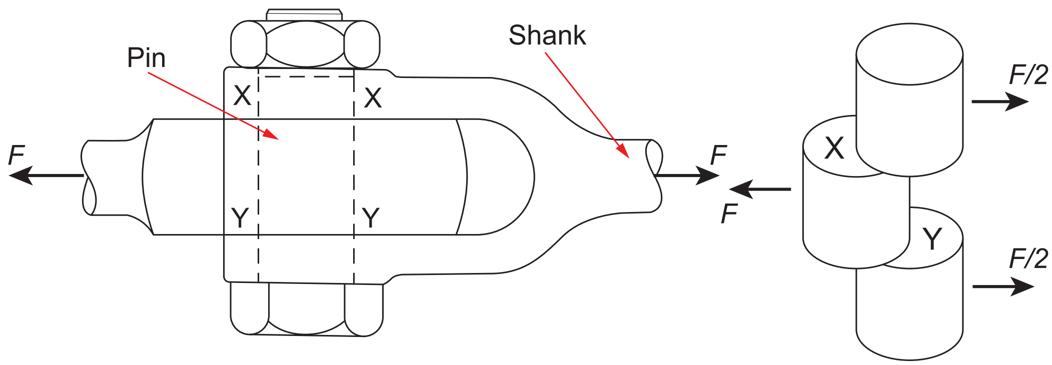
$$A = \frac{\pi d^2}{4}$$

Figure 3 – Single Shear



In Figure 3, the rivet C would only shear across a single cross-sectional plane, x-x, and it is said to be in single shear. In Figure 4, the bolt shown could shear across two cross-sectional planes (X-X and Y-Y). Both these planes resist the shearing force, F. Therefore, both areas are added together, and then used in the formula for stress. Because two planes resist the shear, the bolt is said to be in double shear.

Figure 4 – Double Shear



Example 3

A tie rod made up of two parts, as shown in Figure 4, is to carry a tensile load of 350 kN. Determine the minimum diameter for the connecting bolt if the allowable working stress in shear is limited to 300 MPa.

Solution 3

The bolt in this case is in double shear. The area resisting shear is two times the cross sectional area of the bolt.

$$A = \frac{\pi d^2}{4} \times 2$$

$$= 0.5\,\pi\,d^2$$

$$\tau = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{350 \times 10^3\ \text{N}}{0.50\pi d^2 \text{m}^2}$$

Since the allowable stress, τ, is known, it can be placed into the equation which can be solved to determine the diameter. This value of d will be the minimum diameter required to keep the stress below the allowable 300 MPa.

$$\text{Allowable stress } (\sigma) = \frac{\text{Load}}{\text{Area}}$$

$$300 \times 10^6 \text{N/m}^2 = \frac{350 \times 10^3 \text{ N}}{0.5\ \pi d^2 \text{m}^2}$$

$$0.5\pi d^2 = \frac{350 \times 10^3 \text{ N}}{300 \times 10^6 \text{ N/m}^2}$$

$$d^2 = \frac{350 \times 10^3 \text{ N}}{0.5 \times 3.1416 \times 300 \times 10^6 \text{ N/m}^2}$$

$$d^2 = \frac{350\ 000 \text{ N}}{4.7124 \times 10^8 \text{ N/m}^2}$$

$$d^2 = 7.43 \times 10^{-4} \text{m}^2$$

$$\sqrt{d^2} = \sqrt{7.43 \times 10^{-4} \text{m}^2}$$

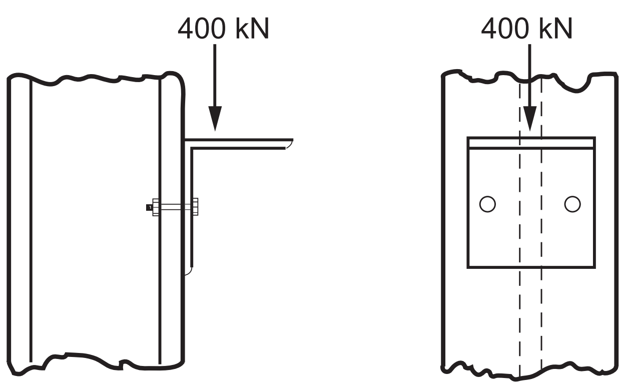
$$d = 0.0273 \text{ m}$$

$$= \mathbf{27.3\ mm\ (Ans.)}$$

Example 4

An angle bracket on a steel column (Figure 5) supports a load of 400 kN. The bracket is bolted to the column with two 16 mm diameter bolts. Find the shear stress in each bolt.

Figure 5 – Angle Bracket



Solution 4

In this case, the bolts are in single shear, but there are two of them. The diameter of each bolt is 16 mm or 0.016 m. The cross-sectional area of each bolt is:

$$A = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0.016^2 \text{ m}^2}{4}$$

$$= \frac{3.1416 \times 0.016^2 \text{ m}^2}{4}$$

$$= \frac{3.1416 \times 0.000\,256 \text{ m}^2}{4}$$

$$= \frac{0.000\,804 \text{ m}^2}{4}$$

$$= \mathbf{0.000\,201\ m^2\ (Ans.)}$$

Assume the load of 400 kN is shared equally by the two bolts. Therefore, the load carried by each bolt is:

$$P = 200 \text{ kN}$$

The average shear stress in each bolt is:

$$\tau = \frac{P}{A}$$

$$= \frac{200 \text{ kN}}{0.000\,201 \text{ m}^2}$$

$$= 995\,025 \text{ kN/m}^2$$

$$= 995\,025 \text{ kPa}$$

$$= \mathbf{995\ MPa\ (Ans.)}$$

Self-Test 1

A square hole with 10 mm sides is to be punched out of a plate that is 9 mm thick. The shear stress of the material is 300 MPa. What force is required on the punch?

108 kN (Ans.)

Terms Related to Stress

Ultimate Stress

The ultimate stress or strength of a material is the stress at the instant it breaks. It is based on the force required to break the material and its original cross-sectional area.

Allowable Stress

The allowable stress or working stress is the amount of stress that a material is allowed to carry. This stress is usually well within the elastic limit for safety reasons.

Elastic Limit

The elastic limit of a material is the maximum stress to which a material can be subjected and still return to its original size and shape when the deforming load is removed.

Maximum Allowable Load

The maximum allowable load is the greatest load that can be applied to a material under its design conditions. It is the load which corresponds to the allowable stress.

Factor of Safety

The factor of safety is the ratio of the ultimate stress to the allowable stress.

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Allowable stress}}$$

When designing a component, the value of the factor of safety is selected after considering several factors, including:

- Mechanical properties of the material
- Loading conditions (e.g. steady, shock, vibration)
- Possibility of corrosion or erosion
- Degree of safety required

Objective 3

Calculate the strain of members under load.

Strain

Strain is a measure of the deformation produced in a member by a load. Normal stresses (tensile or compressive) produce a change in length in the direction of the stress. If Δl is the change in length of a member of original length (L), then:

A S

$$\text{Strain } (\varepsilon) = \frac{\Delta l}{L}$$

Strain is defined as the change in length per original length and thus has no units. However, it is usually expressed in units of metre per metre or millimetre per millimetre (i.e. consistent units).

Tensile stress will cause an increase in length, while compressive stress will decrease the length. Strain is normally considered positive for an increase in length and negative for a decrease in length.

Example 5

A steel bar 2 m long shortens by 4 mm under a compressive load. What will be the strain?

Solution 5

$$\begin{aligned}\text{Strain } (\varepsilon) &= \frac{\Delta l}{L}\\ &= \frac{-4\text{ mm}}{2000\text{ mm}}\\ &= \mathbf{-0.002\ (Ans.)}\end{aligned}$$

Self-Test 2

The strain of a 2.3 metre long boiler through-stay measures 0.00025. How much did the stay grow in length?

0.575 mm (Ans.)

Chapter Summary

In any design process, choosing the best material for the price is very important. In order to choose the right material, one must consider its mechanical properties, including those properties discussed in this chapter.

A load applied to a body will cause an internal resistance that is defined as stress. Materials have their own inherent stress limit. When the resulting stress due to an external load exceeds that limit, the material will break.

When a material is put under load, the stress will cause deformation, known as strain. All forms of stress, tension, compression, and shear, will cause strain. It is up to the designer to make sure that the deformations are within the safe limit.

In engineering, safety factors must be applied. Safety factors remove or minimize potential risk, and compensate for approximations in calculations. In the real world, all stress-strain calculations used for design purposes will have some sort of a safety factor.

Chapter 9

Power Transmission

Learning Outcome

When you complete this chapter you should be able to:

Perform calculations pertaining to common power transmission systems.

Learning Objectives

Here is what you should be able to do when you complete each objective:

1. *Calculate pulley speeds, transmitted power, and efficiency of belt drive systems.*
2. *Calculate gear speeds for gear and chain drive systems.*

Chapter Introduction

Belts, chains, or gears are used to transmit mechanical power, from where the power is developed to where it is used.

The automobile demonstrates many forms of power transmission very well. The combustion engine is the main source of power for a vehicle. However, it is the transmission, through drive shafts, that provides power delivery to the wheels. The transmission contains multiple gears to provide the drive ratios that permit the vehicle to travel at speeds independent of the engine rotational speeds.

Though the power produced by the engine is primarily used to drive the vehicle, it is also used to drive important accessories, such as the alternator, the water pump and the air conditioning compressor. These auxiliaries are commonly driven with belts and pulleys.

This chapter will cover basic power transmission through pulley, chain and gear drive systems. Also discussed will be the inefficiencies that occur in such systems.

Objective 1

Calculate pulley speeds, transmitted power, and efficiency of belt drive systems.

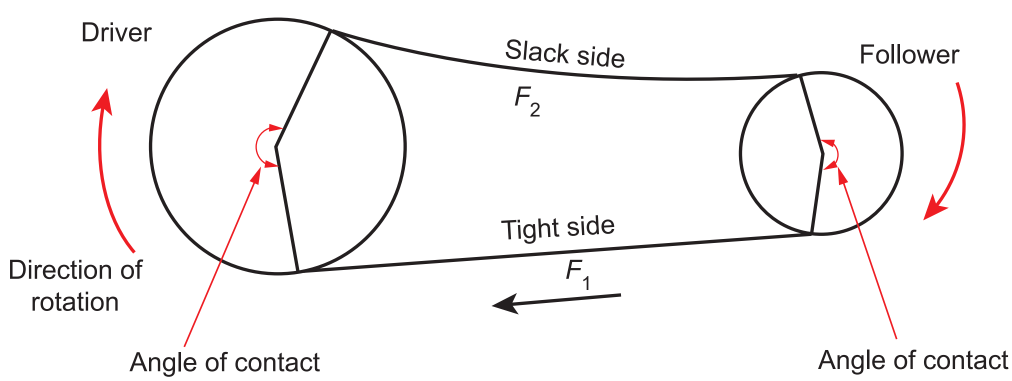
Belt Drives

A belt drive moves a pulley (follower or driven pulley) from another pulley (driver) by means of a friction force between the surface of the belt and the pulleys. Belt drives are used to transmit power, change rotational speeds, or both.

If there is no slippage between the belt and the pulleys, the linear speed of a point on each pulley will be the same and equal to the linear speed of the belt. If the pulleys are the same diameter, the rotational speeds (rev/min) of the pulleys will be equal. If the driver pulley is smaller in diameter than the follower, the follower will rotate at a lower rev/min than the driver will. The opposite is also true.

Consider the belt driven pulley shown in Figure 1. The belt drive should be arranged so that the tension (force F_1) in the bottom belt is always greater than the tension (force F_2) in the top belt. This increases the angle of contact of the belts with the pulleys and reduces slippage, with a resulting increase in the power transmitted from the driver to the follower.

Figure 1 – Belt Driven Pulley

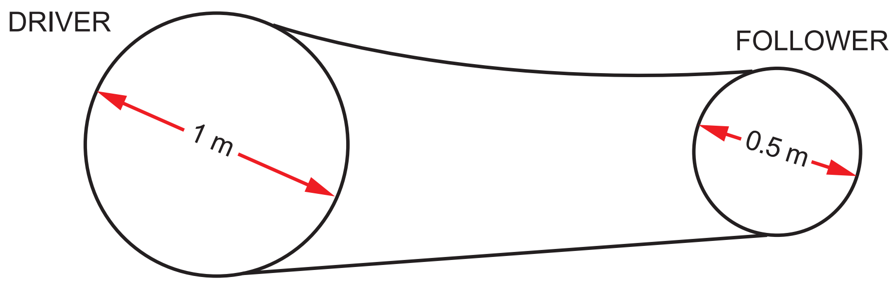


Rotational Speed of Pulleys

If the belts do not slip (efficiency = 100%), then any point on the rim of a pulley will travel at the same linear speed (m/s) as the belt itself. However, the rotational speed (rev/min) of the pulleys will vary if they are different diameters (Figure 2).

For each revolution of the driver, if there is no belt slippage, any point on the belt will travel a distance equal to the circumference of the driver.

Figure 2 – Belt Drive



Referring to Figure 2, for one revolution of the driver, the distance moved by any point on the belt is:

$$\begin{aligned} \text{Distance} &= \pi \text{ D} \\ &= \pi \times 1 \text{ m} \\ &= 3.1416 \text{ m} \end{aligned}$$

This would be the distance traveled by a point on the belt and by a point on the rim of the follower.

For one revolution of the follower:

$$\begin{aligned} \text{Circumference} &= \pi \text{ D} \\ &= \pi \times 0.5 \text{ m} \\ &= 3.1416 \times 0.5 \text{ m} \\ &= 1.5708 \text{ m} \end{aligned}$$

But for one revolution of the driver, a point on the rim of the follower moves 3.1416 m. The follower rotates:

$$3.1416 \text{ m}/1.5708 \text{ m} = 2 \text{ times for each revolution of the driver}$$

Since π is a constant, the revolutions of the pulleys are inversely proportional to the diameters. In the last case:

$$\begin{aligned} \text{Driver diameter} &= 1 \text{ m} \\ \text{Follower diameter} &= 0.5 \text{ m} \\ \text{Revolutions of follower} &= 1 \text{ m}/0.5 \text{ m} \\ &= 2 \times \text{revolution of driver} \end{aligned}$$

Summary

$$\text{If } D_1 = \text{Diameter of pulley 1}$$
$$N_1 = \text{rev/min of pulley 1}$$
$$D_2 = \text{Diameter of pulley 2}$$
$$N_2 = \text{rev/min of pulley 2}$$

Then $$D_1N_1 = D_2N_2$$

Or $$\frac{N_1}{N_2} = \frac{D_2}{D_1}$$

Also, the linear speed of a belt being driven without slippage by a rotating drive wheel of radius R (in m) is:

$$\text{Linear speed (m/s)} = \frac{\text{rev/min} \times \pi D \text{ m/rev}}{60 \text{ s/m}}$$

Example 1

A pulley with a diameter of 1 m is driven at (a) 200 rev/min, (b) 150 rev/min, (c) 120 rev/min. What is the linear speed in m/s of a point on the rim of the pulley for each rotational speed?

Solution 1

a) $$\text{Linear speed (m/s)} = \frac{\text{rev/min} \times \pi D \text{ m/rev}}{60 \text{ s/m}}$$
$$= \frac{200 \text{ rev/min}}{60 \text{ s/min}} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= 3.33 \text{ rev/s} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= \mathbf{10.46 \text{ m/s (Ans.)}}$$

b) $$\text{Linear speed (m/s)} = \frac{\text{rev/min} \times \pi D \text{ m/rev}}{60 \text{ s/m}}$$
$$= \frac{150 \text{ rev/min}}{60 \text{ s/min}} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= 2.5 \text{ rev/s} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= \mathbf{7.85 \text{ m/s (Ans.)}}$$

c) $$\text{Linear speed (m/s)} = \frac{\text{rev/min} \times \pi D \text{ m/rev}}{60 \text{ s/m}}$$
$$= \frac{120 \text{ rev/min}}{60 \text{ s/min}} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= 2 \text{ rev/s} \times 3.1416 \times 1.0 \text{ m/rev}$$
$$= \mathbf{6.28 \text{ m/s (Ans.)}}$$

Example 2

A pulley with a diameter of 1.5 m is driven at 50 rev/min by a belt drive from a pulley of 0.5 m diameter. What is the rev/min of the driving pulley?

Solution 2

$D_1 = 1.5\text{ m}$

$N_1 = 50\text{ rev/min}$

$D_2 = 0.5\text{ m}$

$$\frac{N_1}{N_2} = \frac{D_2}{D_1}$$

$$N_2 = \frac{N_1 D_1}{D_2}$$

$$= \frac{50\text{ rev/min} \times 1.5\text{ m}}{0.5\text{ m}}$$

$$= \frac{75\text{ rev/min/m}}{0.5\text{ m}}$$

= **150 rev/min (Ans.)**

Self-Test 1

A pulley with a diameter of 12 cm drives another pulley with a diameter of 30 cm. The driver rotates at 3450 rev/min. Calculate the rev/min of the driven pulley, and the linear speed of the belt in m/s (assume the belt does not slip).

1380 rev/min, 21.7 m/s (Ans.)

Power Transmitted by Belts

$$\text{Power transmitted (watts)} = (F_1 - F_2) \times \text{Speed of belt}$$

$$\text{Where } F_1 = \text{Tension on tight side (N)}$$

$$F_2 = \text{Tension on slack side (N)}$$

$$\text{Speed of belt} = \text{Speed, m/s}$$

Example 3

The tensions in the tight and slack side of a belt drive system are 2000 N and 500 N, respectively. If the belt speed is 5 m/s, what power will be transmitted if there is no belt slippage?

Solution 3

$$\begin{aligned}
\text{Power transmitted (watts)} &= (F_1 - F_2) \times \text{Speed of belt} \\
&= (2000\text{ N} - 500\text{ N}) \text{ x } 5\text{ m/s} \\
\text{Power} &= 1500\text{ N} \times 5\text{ m/s} \\
&= 7500\text{ Nm/s} \\
&= 7500\text{ J/s} \\
&= 7500\text{ W} \\
&= \mathbf{7.5\ kW\ (Ans.)}
\end{aligned}$$

Belt Slippage

In practice, belts slip on pulley drives. This results in a loss of energy or power transmitted, so that the efficiency is less than 100%.

$$\%\text{ Efficiency} = \frac{\text{Power output}}{\text{Power input}} \times 100\%$$

Example 4

In a belt drive system, the input to the driver pulley is 75 kW and the output from the follower is 70 kW. What is the % efficiency of the drive?

Solution 4

$$\begin{aligned}
\%\text{ Efficiency} &= \frac{\text{Power output}}{\text{Power input}} \times 100\% \\
&= \frac{70\text{ kW}}{75\text{ kW}} \times 100\% \\
&= 0.9333 \times 100\% \\
&= \mathbf{93.33\%\ (Ans.)}
\end{aligned}$$

Self-Test 2

The tensions in the tight and slack side of a belt drive system are 1850 N and 525 N, respectively. If the belt speed is 8 m/s, what power will be transmitted if there is no belt slippage? If the efficiency is 92%, what is the power output from the follower? Give both answers in kW.

10.6 kW, 9.75 kW (Ans.)

Pulley Trains

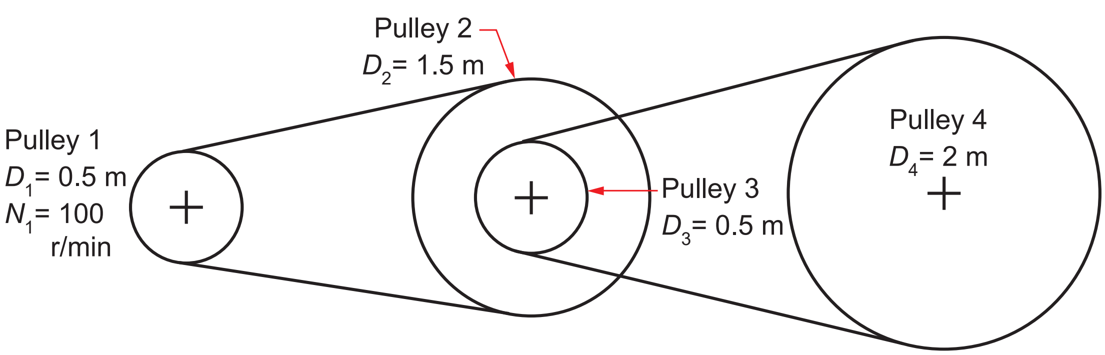
A pulley train consists of a series of pulleys connected by belts. Pulley trains are used to change speeds and/or to transmit power at varying speeds to different applications. When considering pulley trains, the principles for simple pulley arrangements can be applied.

Example 5

Find the linear speed of the 2 m diameter pulley in the train shown in Figure 3 if there is:

a) No belt slippage
b) Overall belt slippage of 10%

Figure 3 – Pulley Train



Solution 5

a) If N_1 = 100 rev/min

$$N_2 = N_1 \times \frac{D_1}{D_2}$$

$$= 100 \text{ rev/min} \times \frac{0.5 \text{ m}}{1.5 \text{ m}}$$

$$= 100 \text{ rev/min} \times 0.3333$$

$$= 33.33 \text{ min}$$

If pulley #2 rotates at 33.33 rev/min, then pulley #3 must also rotate at 33.33 rev/min, since it is on the same shaft. Therefore, $N_2 = N_3$ = 33.33 rev/min

$$N_3 \times D_3 = N_4 \times D_4$$

$$N_4 = \frac{N_3 \times D_3}{D_4}$$

$$= \frac{33.33 \text{ rev/min} \times 0.5 \text{ m}}{2\text{m}}$$

$$= 8.33 \text{ rev/min}$$

Thus

$$\text{Linear speed of Pulley 4} = N_4 \times \pi \times D_4 = 8.33 \text{ rev/min} \times 3.1416 \times 2 \text{ m/r}$$

$$= 52.34 \text{ m/min}$$

$$= \frac{52.34 \text{ m/min}}{60 \text{ sec/min}}$$

$$= \mathbf{0.87 \text{ m/s (Ans.)}}$$

b) Assuming 10% belt slippage overall:

$$\text{Linear speed of Pulley 4} = 0.87 \text{ m/s} - (0.10 \times 0.87 \text{ m/s})$$

$$= 0.87 - 0.087 \text{ m/s}$$

$$= \mathbf{0.783 \text{ m/s (Ans.)}}$$

Note: With belt drives, the pulleys all rotate in the same direction.

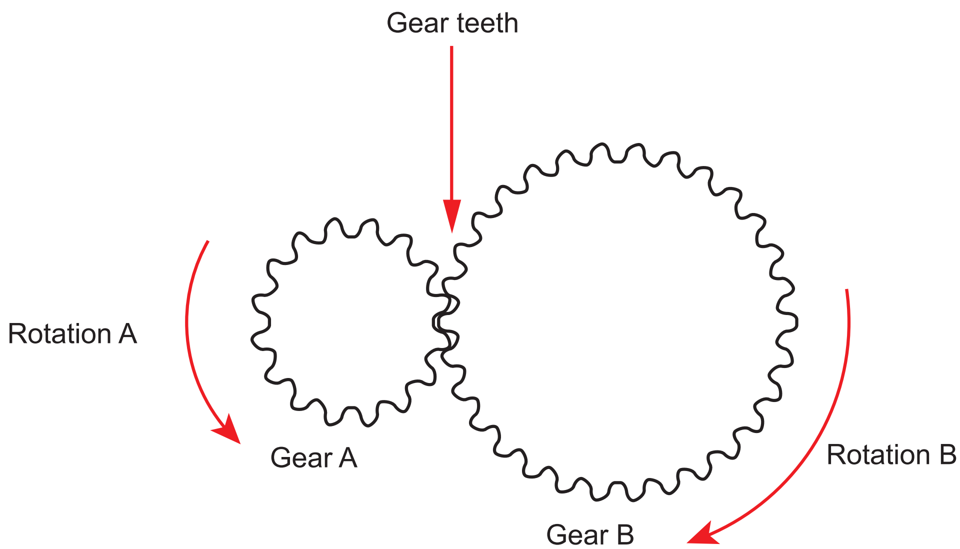
Objective 2

Calculate gear speeds for gear and chain drive systems.

Gear Drives

In gear drives, the teeth in mating gear wheels mesh together to transmit power from one to the other. For single gears meshing together, the direction of rotation becomes reversed, as shown in Figure 4.

Figure 4 – Gear Drive



The ratio of the rotational speeds of the gears is inversely proportional to the number of teeth on each gear, such that (referring to Figure 4):

$$\frac{\text{Rotational speed A (rev/min)}}{\text{Rotational speed B (rev/min)}} = \frac{\text{Number of teeth on B}}{\text{Number of teeth on A}}$$

Example 6

A gear wheel having 20 teeth and rotating at 200 rev/min drives a gear that has 40 teeth. What will be the rev/min of the driven gear?

Solution 6

$$\text{Speed of driven gear} = \text{speed of driver gear} \times \frac{\text{Number of teeth on driver gear}}{\text{Number of teeth on driven gear}}$$

$$= 200 \text{ rev/min} \times \frac{20 \text{ teeth}}{40 \text{ teeth}}$$

$$= 200 \text{ rev/min} \times 0.5$$

$$= \mathbf{100\ rev/min\ (Ans.)}$$

Intermediate Gears

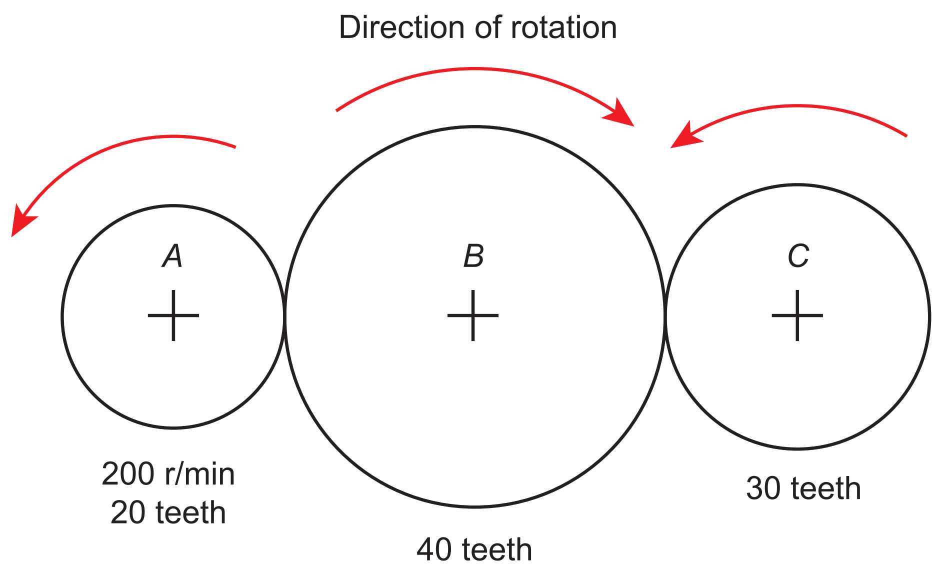
Intermediate gears, also called idler gears, are used to transmit power between gears that are some distance apart. Intermediate gears do not affect the speed of the driven gear, since the same number of teeth mesh with both the driven gear and driver.

If two intermediate gears were used, then the rotation of the driver and driven gears would be in opposite directions. The intermediate gears do not affect the speed ratio.

Example 7

An idler gear is used as shown in Figure 5. The driver rotates at 200 rev/min and has 20 teeth. If the driven gear has 30 teeth and the idler gear has 40 teeth, what will be the rev/min of the driven gear?

Figure 5 – Idler Gear



Solution 7

$$\text{rev/min B} = \text{rev/min A} \times \frac{\text{Number of teeth on A}}{\text{Number of teeth on B}}$$

$$= 200 \text{ rev/min} \times \frac{20 \text{ teeth}}{40 \text{ teeth}}$$

$$= 200 \text{ rev/min} \times 0.5$$

$$= 100 \text{ rev/min}$$

$$\text{rev/min C} = \text{rev/min B} \times \frac{\text{Number of teeth on B}}{\text{Number of teeth on C}}$$

$$= 100 \text{ rev/min} \times \frac{40 \text{ teeth}}{30 \text{ teeth}}$$

$$= 100 \text{ rev/min} \times 1.3333$$

$$= \mathbf{133.33 \text{ rev/min (Ans.)}}$$

If there was no intermediate gear:

$$\text{rev/min C} = \text{r/min A} \times \frac{\text{Number of teeth on A}}{\text{Number of teeth on C}}$$

$$= 200\ \text{rev/min} \times \frac{20\ \text{teeth}}{30\ \text{teeth}}$$

$$= 200\ \text{rev/min} \times 0.6667$$

$$= \mathbf{133.33\ rev/min\ as\ before}$$

The idler gear does cause gear C to rotate in the same direction as A. The advantage of gear drives over belt drives is that there is no slippage. However, badly meshing or poorly lubricated gears can cause excessive friction and low efficiency. Gears are able to transmit more power than belt systems of comparable size.

Self-Test 3

An 18 tooth gear drives an idler gear with 54 teeth. The idler gear drives a 43 tooth gear. If the 18 tooth gear rotates at 300 rev/min, find the rev/min of the idler gear, and the rev/min of the 43 tooth gear.

100 rev/min, 125.6 rev/min (Ans.)

Gear Trains

A number of gears may mesh together in a train, to produce variations in shaft speeds and directions of rotation. Gear trains are considered in a similar manner to pulley trains.

Backlash

Because clearance is necessary between teeth that have to mesh, gears can be moved slightly when not driving. This movement is called backlash and, if excessive, can cause extreme forces on the teeth during starting or reversing.

Chain Drives

Chain drives use special gears called sprockets, which are driven by chains. An advantage of chain drives is that the gears do not have to mesh together, and a positive drive can be obtained over a longer distance. The speeds of chain drives are calculated in the same way as for gear drives.

Chapter Summary

This chapter discussed the two different forms of power transfer systems. A pulley system will share an inherent relationship between the rotation speed and diameter size of each pulley. As is the case with any form of power transfer, inefficiencies will always exist. Most modern day power transfer systems are very efficient.

However, no system is perfect and belt slippage does occur. When a belt slips, the output power will be less than the input power, resulting in less than 100% power transfer. Here is a hint for solving problems involving pulley trains: two pulleys attached to the same shaft, no matter what diameter they may be, will spin at the exact same speed.

Power transfer and pulley systems are related by their diameter size and rotational speed. Gear drives are related by the number of teeth, and by their rotational speed. Idler gears may be used to provide spacing between two gears without altering the speed at which the driven gear operates. A single idler gear can also be used to cause both driven and driver gear to spin in the same direction.

Unit Summary

This ends Unit 1 on Elementary Mechanics. The following is a short summary of each chapter and serves as a reminder of all the material covered throughout this unit.

Chapter 1: This chapter was about the concept of force and how it relates to acceleration. It touched upon the idea of work, and how work is related to force. Energy and power were introduced. Energy was described as the ability to do work, which also means energy is the ability to apply a force over a distance. Force is an ongoing concept throughout the unit. This chapter was a summary of what was to come in the following chapters and serves as a foundation for the rest of the unit.

Chapter 2: Continuing from the introduction of force in Chapter 1, the concept of a moment was introduced. In order for an object to be statically balanced, it must be at equilibrium. That means that all the forces and moments are in balance. A moment is the result of a force acting a perpendicular distance away from a point of rotation.

Chapter 3: The use of moments helps to gain mechanical advantage, and to get large output forces from small input forces. In order to do this, use simple machines. However, simple machines are always less than 100% efficient.

Chapter 4: A force has two components. One component is the magnitude. The other is the direction in which the force is applied. Therefore, a force is a vector. Time, on the other hand, only has magnitude; therefore, it is a scalar. This chapter covered the concept of scalars and vectors.

Chapter 5: When an object has a net force acting on it that is greater than zero, it must accelerate. If it is accelerating, then its velocity is increasing. This chapter covered how force, acceleration, and velocity are related. It summed up the different expressions used to relate displacement, velocity, and acceleration. It also showed how important graphical representation can be. Sometimes a Power Engineer will struggle to solve a problem without visual aids. Graphs are an excellent means of visualizing a problem.

Chapter 6: Force applied over a distance is termed as work. In order to do work, there must be the ability to do so; this is known as energy. Energy is the fundamental reason why power plants exist. An important concept introduced in this chapter was the difference between gauge and absolute pressure.

Chapter 7: When an object is placed on a surface and dragged across it, there is a resistance towards the motion of the object. This resistance is a result of the contact between the surface and the object; this is defined as friction. This chapter covered the different types of friction and the laws governing them. Friction is proportional to the normal resulting force and is dependent on the coefficient of friction.

Chapter 8: When a force is applied to a static object - that is all the forces are balanced - the object experiences internal resistance. This internal resistance is defined as stress. When an object is under stress, it slightly deforms, which is strain. Stress causes strain. This chapter covered the different mechanical properties of a material, the different types of stress, and the definition of strain.

Chapter 9: The final chapter of Unit 1 covered the basic forms of transmitting energy with the use of pulleys and gears. Neither system is 100% efficient. Both operate on the principal of energy conservation. Both are dependent on the geometry of the transmitting system: based on pulley diameters or on the number of gear teeth in a gear and chain drive system.

A self-assessment tool is available on MyPower LMS. Login using the unique user ID and password found on the inside front cover of Unit 1.

Notes

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UNIT A-1

Knowledge Exercises

Knowledge Exercises – Chapter 1

Name: ______________________________ Date: ________________________________

Instructor: ___________________________ Course: ______________________________

Objective 1

1. A body travels 1000 m in 65 seconds. What is its average velocity?

2. A body moved north at 15 m/s. After 7 seconds, it had a velocity of 40 m/s in the same direction. What was the acceleration of the body?

3. A mass of 250 kg is accelerated at 17 m/s^2. What force is required to achieve this acceleration?

4. A body with a mass of 75 kg is allowed to fall freely to the ground. What gravitational force is acting on the mass?

Objective 2

5. A hydraulic plunger has an area of 0.10 m^2 and a force of 200 N acts on it. What is the pressure exerted on the plunger?

6. Give the definition of potential energy.

Knowledge Exercises – Chapter 2

Name: ______________________________ Date: ______________________________

Instructor: ______________________________ Course: ______________________________

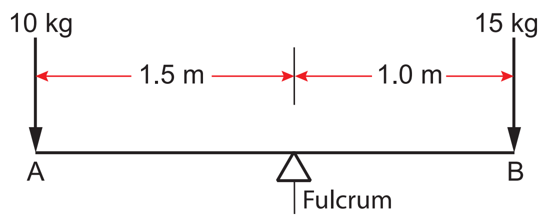
Objective 1

1. Define the term "moment of a force".

2. List the conditions when the forces of a system are said to be in equilibrium.

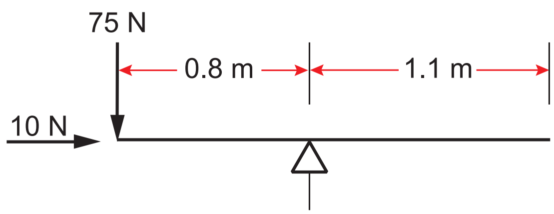
Objective 2

3. In the diagram shown below, what forces act on the lever? Is it in equilibrium?

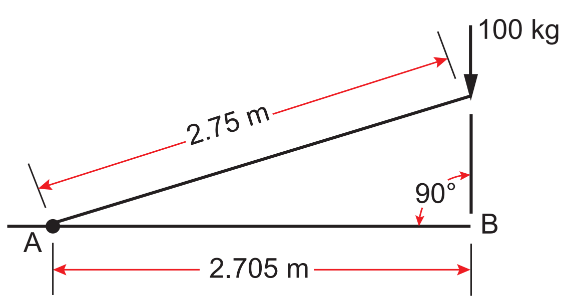


Chapter 2 (Cont.)

4. What forces are required at the right hand end so that the bar is in equilibrium?



5. What is the moment of the force about Point A due to the 100 kg mass?



Knowledge Exercises – Chapter 3

Name: ______________________________ Date: ________________________________

Instructor: ___________________________ Course: ______________________________

Objective 1

1. Give the definition of Actual Mechanical Advantage (MA) of a simple machine.

2. Give the definition of Velocity Ratio (VR) of a simple machine.

3. Give a brief description of how the efficiency of a simple machine is determined.

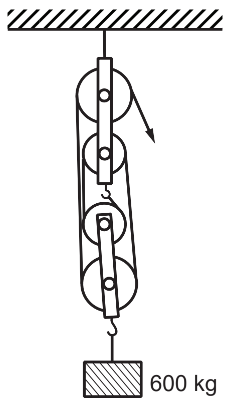
4. The velocity ratio of a machine is 10. The machine lifts 2000 N a distance of 2 m, with an effort of 400 N. Find the following:
   a) Mechanical advantage
   b) % Efficiency
   c) Distance moved by the effort

Chapter 3 (Cont.)

5. In the pulley block assembly shown below, assuming a system efficiency of 95%, find the following:

   a) Force to raise the load.

   b) How high does the load rise if the effort moves 4 m?



6. What is the work input per person if 3 people are required to roll a 400 kg barrel up a ramp 6 m long, to raise it 1.5 m? The roughness of the ramp's surface increases the work required by 15%.

7. Explain how the velocity ratio of a pulley block system is determined.

Knowledge Exercises – Chapter 4

Name: ______________________________ Date: ________________________________

Instructor: ___________________________ Course: ______________________________

Objective 1

1. Define the term "scalar quantity."

2. Define the term "vector quantity."

3. Draw a space diagram for three coplanar forces acting outward from a single point. The forces are 60 N due east, 40 N 35° north of east, and 80 N 30° east of south.

4. Add the following vectors and determine the length and the angle from the origin.

   a) 25 N, 45° + 45 N, 35°

   b) 80 N, 60° + 20 N, 10°

   c) 30 m/s, 120° + 50 m/s, 285°

   d) 85 km/h, 80° + 110 km/h, 140°

Knowledge Exercises – Chapter 5

Name: ______________________________ Date: ________________________________

Instructor: ___________________________ Course: ______________________________

Objective 1

1. A train is traveling at 90 km/h. Find the following:

   a) Its speed in m/s

   b) How far it will travel in 7 seconds

   c) How long it takes to cover 4 km

   d) The average speed in km/h, if the time to travel 130 km is one hour and 40 minutes

Objective 2

2. A vehicle starts from rest and reaches a velocity of 100 km/h in 4 minutes. If the velocity increased uniformly, how far will the vehicle travel during this time? Check calculations using a velocity-time graph. Give the answer in km.

Chapter 5 (Cont.)

Objective 3

3. A car traveling at a speed of 140 km/h decelerates for 20 seconds. During this time, it travels 500 m.

   a) What is its deceleration, in m/s^2?

   b) What is its speed after the deceleration, in km/h?

Objective 4

4. A train starts from rest and increases its speed uniformly for three minutes. In that time, it reaches a speed of 150 km/h. What is its acceleration in m/s^2?

5. A car traveling at an initial speed of 40 km/h accelerates uniformly to 120 km/h in two minutes. What distance is travelled in km during the acceleration period?

6. Give the definition of acceleration.

Knowledge Exercises – Chapter 6

Name: ______________________ Date: ______________________

Instructor: ______________________ Course: ______________________

Objective 1

1. A hoist system raises a mass of 750 kg a vertical distance of 80 m. Calculate the work done and the quantity of work input to the system if the hoist is 85% efficient.

2. If the hoist in Question 1 raises the load in 10 seconds, what is the power developed and what is the power input?

Objective 2

3. A cylindrical tank, 1 m in diameter, has flat ends. What force is exerted against each end if a pressure of 350 kPa is contained in the tank?

Objective 3

4. A 75 kg boiler drum manhole cover falls from the top of the boiler to the boiler room floor. If this distance is 25 m, what is its velocity at the point of impact?

5. Give the definitions for kinetic energy and potential energy.

6. River water with a mass of 100 tonnes is moving at 12.5 km/h. What is its kinetic energy?

Knowledge Exercises – Chapter 7

Name: ______________________________ Date: ________________________________

Instructor: ___________________________ Course: ______________________________

Objective 1

1. Define "force of friction".

2. Explain how lubrication reduces sliding friction.

Objective 2

3. Define "coefficient of friction".

4. Find the coefficients.

   a) Find the coefficient of static friction between a crate and a wooden floor if the crate has a mass of 400 kg and the horizontal force applied is 1600 N.

   b) Find the coefficient of kinetic friction if a force of 1400 N is required to keep the same crate moving at constant speed.

Chapter 7 (Cont.)

c) Find the coefficient of kinetic friction between the same crate and the floor if water is used as a lubricant and the force required moving the crate at a constant speed is 1100 N.

5. Find the force required to keep a wooden block moving along a horizontal surface if the block has a mass of 25 kg and the coefficient of friction between the two surfaces is 0.19.

Knowledge Exercises – Chapter 8

Name: ______________________ Date: ______________________

Instructor: ______________________ Course: ______________________

Objective 1

1. Define the following terms in your own words.

   a) Hardness

   b) Brittleness

   c) Ductility

   d) Elasticity

Objective 2

2. A solid cylinder with an outside diameter of 100 mm supports a load of 100 kN. What is the compressive stress in the material?

3. A tie bar is made of a material having an ultimate tensile strength of 231 MPa and must carry a load of 255 kN. What is the diameter of the bar if a factor of safety of 7 is required?

Chapter 8 (Cont.)

4. A round steel bar, 5 cm in diameter, is placed in tension by a load of 1 tonne. What is the stress in the bar?

Objective 3

5. A bar 36.5 mm in diameter and 1.5 m long stretches 1.2 mm when a load of 240 kN is applied. What are the stress and strain?

Knowledge Exercises – Chapter 9

Name: ______________________ Date: ______________________

Instructor: ______________________ Course: ______________________

Objective 1

1. A belt pulley has a diameter of 0.855 m and is driven at 305 rev/min.

   a) What is the linear speed in m/s of a point on the rim of the pulley?

   b) If the pulley hub is 200 mm in diameter, what is the linear speed of a point on the hub?

2. The tension in the tight side of a belt is 2500 N and in the slack side it is 475 N. The drive pulley is 0.9 m in diameter and turns at 150 rev/min. If the efficiency of the drive is 95%, what is the:

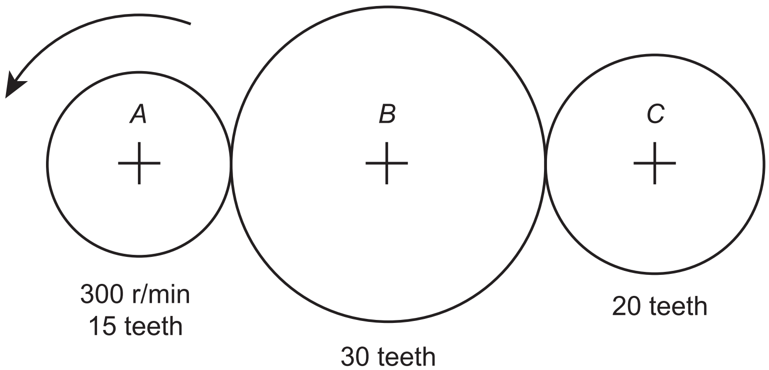
   a) Power input

   b) Power output

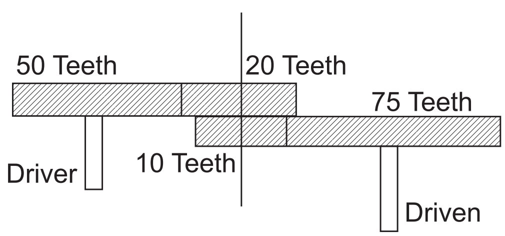
Chapter 9 (Cont.)

Objective 2

3. In the gear train shown below, the driver gear (A) is rotating counterclockwise at 300 rev/min and has 15 teeth. The drive gear (A) meshes with the intermediate gear (B) having 30 teeth, which meshes with the driven gear (C) having 20 teeth. Find the rotational speed and the direction of rotation for the intermediate gear (B) and the driven gear (C).



4. The gear train shown has a driver speed of 150 rev/min in a clockwise direction. Find the speed of the driven shaft and its direction of rotation.



5. A chain drive has a drive sprocket of 300 mm diameter and a driven sprocket of 1000 mm diameter. What is the speed in rev/min of the driven shaft if the driver shaft turns at 890 rev/min?

Additional Workspace

Additional Workspace

Additional Workspace

Additional Workspace